Please do not ask questions during this exam. If you consider a question to be ambiguous, state your assumptions in the margin and do the best you can to provide the correct answer. Refer to this page and the next for formulas, general directions, and calculator troubleshooting tips.

- Any communication with any person (other than the instructor or the designated proctor) during this exam in any form, including written, signed, verbal or digital, is understood to be a violation of academic integrity.
- All devices, such as computers, cell phones, cameras and PDAs must be turned off while the student is in the testing room.
- The only calculators to be used are TI-83, TI-83+, TI-84 or TI-84+. You may NOT borrow or share a calculator with another person taking this test.
- Statement of Academic Integrity: I have not and will not give or receive improper aid on this test.

In signing below, I acknowledge that I have read, understand, and agree to these testing conditions.

Student’s Signature: _________________________________

(This test will not be accepted for grading unless it bears the signature of the student.)

<table>
<thead>
<tr>
<th>FR#1</th>
<th>FR #2</th>
<th>FR #3</th>
<th>FR #4</th>
<th>FR #5</th>
<th>FR#6</th>
<th>Free Response Total</th>
<th>Multiple Choice Total</th>
<th>Total Points Earned</th>
</tr>
</thead>
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<tr>
<td>7</td>
<td>11</td>
<td>6</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>32</td>
<td>68</td>
<td>100</td>
</tr>
</tbody>
</table>

Useful Formulas:

\[
F(t) = P(1 + rt)
\]

\[
F(t) = Pe^r
\]

\[
F(t) = P\left(1 + \frac{r}{n}\right)^n
\]

\[
APY = (e^r - 1)100\% \quad \text{or} \quad APY = \left[\left(1 + \frac{r}{n}\right)^n - 1\right]100\%
\]
General Directions:

- Show work where possible. Answers without supporting work (where work is appropriate) may receive little credit.

- **Do not round intermediate calculations.**

- Answers in context ALWAYS require units.

- Round your answers to **3 decimal places** UNLESS the answer needs to be rounded differently to make sense in the context of the problem OR the directions specify another type rounding OR the complete answer has less than 3 decimal places.

- When you are asked to write a model, include all components of a model: an equation, a description of the input including units, a description of the output including units, and the interval when known.

- When asked to write a sentence of practical interpretation, answer the questions: **when?, what?, and how much?** using ordinary, conversational language. DO NOT use math words, terms, or unnecessary phrases.

- Always use a ruler when estimating values off of a graph.

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**HINTS FOR TROUBLESHOOTING YOUR CALCULATOR:**

- If you lose your L1, L2, etc., you may reinsert them using STAT 5 (set-up editor) enter.

- The SCATTER PLOT will not show unless Plot 1 has been turned on and there is data in L1 and L2.

- ZOOM 0 *may* not work for graphing if Plot 1 is turned on.

- DIM MISMATCH error usually means that the lists in L1 and L2 are not of equal length.

- DATA TYPE error usually means that you already have something in Y1 and you need to clear it before you can paste a new equation.

- INVALID DIM error usually means that your plot(s) are on, but that you have no data in the lists. Refer to the second hint above.

- If your batteries die, raise your hand and hold up your calculator. If your instructor has an extra calculator available, he/she will loan it to you for a few minutes.

- SYNTAX ERROR: Try GO TO. This will happen if you use a subtraction minus sign when you should use a negative sign.
MULTIPLE CHOICE: 68 points

Use a #2 pencil and completely fill each bubble on your scantron to answer each multiple choice question. (For future reference, circle your answers on this test paper.) There is no penalty for guessing on multiple choice. If you indicate more than one answer, or you leave a blank, the question will be marked as incorrect. Each question is worth 2 or 3 points, as indicated.

1. \( p(t) \) million people gives the resident population of the United States between 1900 and 2000, \( t \) years since 1900. Write the following statement using function notation. \[ 3 \text{ pts} \]

In 1940, the resident population of the United States was approximately 132,165,000 people.

a. \( p(1940) = 132,165,000 \)  
b. \( p(1940) = 132.165 \)

c. \( p(40) = 132,165,000 \)  
d. \( p(40) = 132.165 \)

2. \( f(x) = 72 + 15e^x - \sqrt{3x} \) satisfies the definition of a function because ___________________.

\( f(x) = 72 + 15e^x - \sqrt{3x} \) has a(n) _____________________________ representation. \[ 2 \text{ pts} \]

a. for each input value, there is exactly one output value; numeric  
b. for each input value, there is exactly one output value; algebraic  
c. for each output value, there is exactly one input value; numeric  
d. for each output value, there is exactly one input value; algebraic

3. Using a function model to make an estimation for an input value that is outside of the data range describes ______________________________. \[ 2 \text{ pts} \]

a. interpolation  
b. an initial value  
c. slope  
d. extrapolation

4. All linear functions have ____________________________ rate of change. \[ 2 \text{ pts} \]

a. increasing  
b. decreasing  
c. constant  
d. oscillating

5. A cubic function has ______________________________. \[ 2 \text{ pts} \]

a. two inflection points and one concavity  
b. one inflection point and two concavities

c. one inflection point and one concavity  
d. two inflection points and two concavities
6. Which one of the following could be describing the end behavior of a cubic function \( f(x) \)?  

\[
\begin{align*}
\text{a. } & \lim_{x \to -\infty} f(x) = \infty \text{ and } \lim_{x \to \infty} f(x) = \infty \\
\text{b. } & \lim_{x \to -\infty} f(x) = -\infty \text{ and } \lim_{x \to \infty} f(x) = -\infty \\
\text{c. } & \lim_{x \to -\infty} f(x) = 0 \text{ and } \lim_{x \to \infty} f(x) = 0 \\
\text{d. } & \lim_{x \to -\infty} f(x) = \infty \text{ and } \lim_{x \to \infty} f(x) = -\infty
\end{align*}
\]

Use the following graph to answer the next two questions.  

7. Which one of the following statements correctly describes the end behavior of the function graphed above?  

Complete the following sentence:  *As input values increase without bound, output values ____________.*  

\[
\begin{align*}
\text{a. } & \text{decrease without bound} \\
\text{b. } & \text{increase without bound} \\
\text{c. } & \text{get closer and closer to } 2 \\
\text{d. } & \text{get closer and closer to } 0.
\end{align*}
\]

8. Give the equations of any vertical or horizontal asymptotes for the graph of the function shown above.  

\[
\begin{align*}
\text{a. } & \text{vertical asymptote: none; horizontal asymptote: none} \\
\text{b. } & \text{vertical asymptote } x = 0; \text{ horizontal asymptote } y = 2 \\
\text{c. } & \text{vertical asymptote } y = 0; \text{ horizontal asymptote } x = 2 \\
\text{d. } & \text{vertical asymptote } x = 0; \text{ horizontal asymptote } y = 0
\end{align*}
\]
Use the following graph to answer the next two questions. [3 pts each]

9. Find the following:
\[ f(2) = \underline{\phantom{0000}} \], \[ \lim_{} f(x) = \underline{\phantom{0000}} \], and \[ \lim_{} f(x) = \underline{\phantom{0000}} \]
   a. 4, 4, and 5, respectively     b. 5, 4, and 5, respectively
   c. 4, 5, and “does not exist”, respectively     d. “undefined”, 4, and 5, respectively

10. On the interval \( x < 0 \), the concavity of the function \( f(x) \) can be described as __________________ and the direction of the function \( f(x) \) can be described as ________________________________.
   a. Concave down; decreasing     b. Concave down; increasing
   c. Concave up; decreasing      d. Concave up; increasing

11. If data has constant, or nearly constant, first differences, then a/an __________________ function is best to model that data. If a data has constant, or nearly constant, percentage change, then a/an ____________________________ function is best to model the data. [2 pts]
   a. exponential; linear     b. logarithmic; exponential
   c. linear; exponential      d. quadratic, linear
12. Consider the graph of the function \( f(x) \) below. The reason that the function is not continuous at \( x = 3 \) is because _____________________. [2 pts]

![Graph of function f(x) with a point at x = 3]

- \( \lim_{x \to 3} f(x) \) does not exist
- \( f(3) \) is undefined
- \( \lim_{x \to 3} f(x) \) does not exist
- \( \lim_{x \to 3} f(x) \neq f(3) \)

13. \( E(x) = 16.317x + 81.567 \) million users gives the number of European email users, \( x \) years since 2000, \( 5 \leq x \leq 10 \). [3 pts]

According to this model, which of the following three statements are correct?

I. Between the years 2005 and 2010, the number of European email users increased by 16.317 million users.
II. \( E(7) \) uses interpolation to estimate the number of European email users in 2007.
III. In 2000, the number of European email users was 81.567 million users.

- a. I and II only
- b. II and III only
- c. I and III only
- d. I, II, and III

14. \( u(x) = 3.356(1.022^x) \) thousand people gives the population of Iron Country, Utah, U.S., \( x \) years since 1900, \( 0 \leq x \leq 110 \). According to this model, between 1900 and 2010, the population of Iron Country ________________ each year. [3 pts]

- a. increased by 2.2%
- b. decreased by 2.2%
- c. increased by 3.356%
- d. decreased by 97.8%
15. The number of US post offices in 1900 was 77,334 thousand post offices. Between 1900 and 2013, the number of US post offices decreased by 0.9% each year. [3 pts]

Complete the following model by finding an exponential function that fits the data.

\[ p(x) = \text{___________} \text{ thousand post offices gives the number of US post offices, } x \text{ years after 1900, } 0 \leq x \leq 113 \, . \]

a. \(-0.991x + 77.334\)  
   b. \(77.334(0.991^x)\)  
   c. \(77.334(1.009^x)\)  
   d. \(77.334x - 0.991\)

16. Consider the function \(f(x) = 2(0.5^x)\). Which one of the following corresponds to the graph of \(f\)? [2 pts]

![Graphs of exponential functions]

a.   
   b.   
   c.   
   d. 

17. Which one of the following is true about the function \(f(x) = 4 + \ln x\)? [2 pts]

a. It has a horizontal asymptote.  
   b. It has a vertical asymptote.  
   c. It has an inflection point.  
   d. All of the above are true.

18. What is the annual percentage yield of an investment paying 1.75 percent compounded quarterly? [2 pts]

a. 1.750%  
   b. 1.760%  
   c. 1.762%  
   d. 1.765%
19. Which of the following is **not always** true of a logistic function? [2 pts]
   a. It has an inflection point.
   b. It is either always increasing or always decreasing.
   c. It has two horizontal asymptotes.
   d. As the input approaches negative infinity, the output approaches zero.

**Use the following to answer the next two questions.** [2 pts each]

An investment of $600 is deposited in an account paying 4.8% APR compounded semiannually.

20. The investment will be worth _____ after 3 years and 3 months, with no further deposits or withdrawals.
   a. $700.00  
   b. $690.61  
   c. $691.75  
   d. $708.36

21. How long does it take for the investment of $600 to double in value?
   a. 14 years and 7 months  
   b. 14 years and 6 months  
   c. 14 years and 9 months  
   d. 15 years

22. The total ticket sales, in billion Euros, at a yearly festival in France between 2002 and 2009 have the following scatter plot: [2 pts]

Which type of function best models the total ticket sales at the festival?
   a. Exponential  
   b. Logistic  
   c. Cubic  
   d. Logarithmic
Use the following to answer the next three questions. [3 pts each]

\( t(x) \) million transistors gives the number of transistors that Intel was able to fit on a computer chip, \( x \) years since 1989.

\( a(t) \) square micrometers gives the average area of a single transistor when there are \( t \) million transistors on a computer chip.

23. These two models can be combined using ________________. The new function has notation _____.
   a. multiplication; \( t(x) \cdot a(t) \)
   b. division; \( t(x) / a(t) \)
   c. composition; \( t(a(x)) \)
   d. composition; \( a(t(x)) \)

24. The input units of the new function are ______ and the output units of the new function are ______.
   a. years; square micrometers
   b. million transistors; square micrometers
   c. years; million transistors
   d. million transistors; years

25. If \( t(x) = 0.992(1.355^x) \) and \( a(t) = \frac{1.5}{t} \), find the average area of a single transistor on a computer chip in 1995.
   a. 0.244 square micrometers
   b. 1.070 square micrometers
   c. 1.535 square micrometers
   d. 1.565 square micrometers

26. \( s(x) \) percent (written in decimal form) gives the percentage of high school students attending a US public school who have never smoked cigarettes, where \( x \) is the number of years since 1993, \( 0 \leq x \leq 14 \).
   \( n(x) \) thousand students gives the number of high school students attending a US public school, \( x \) years since 1993, \( 0 \leq x \leq 14 \).

   How can these two functions be combined to create the following model?

   \( f(x) = \) ________________ thousand students gives the number of high school students attending a US public school who have never smoked cigarettes, \( x \) years since 1993.
   a. \( s(x) + n(x) \)
   b. \( n(x) \cdot s(x) \)
   c. \( n(x) - s(x) \)
   d. \( n(x) / s(x) \)
Use the following to answer the next two questions. [3 pts each]

\[ m(x) = \frac{77.917}{1 + 32.414e^{-0.854x}} \]

gives the percentage of online 30-49 year olds who use social networking sites, \( x \) years since 2004, \( 0 \leq x \leq 9 \).

27. The input units of \( m(x) \) are ______________ and the output units of \( m(x) \) are ______________.
   a. years; percent  
   b. percent; years  
   c. years; social networking sites  
   d. social networking sites; percent

28. The graph of \( m(x) \) has end behavior __________________. The corresponding horizontal asymptote has equation __________________.
   a. \( \lim_{x \to \infty} m(x) = 77.917 \) and \( y = 77.917 \)  
   b. \( \lim_{x \to \infty} m(x) = 0 \) and \( y = 0 \)  
   c. \( \lim_{x \to -\infty} m(x) = 32.414 \) and \( y = 32.414 \)  
   d. \( \lim_{x \to -\infty} m(x) = 0 \) and \( y = 77.917 \)

FREE RESPONSE: 32 points

Show work where possible. Read the directions on rounding, inclusion of units, and writing models and sentences at the front of the test.

1. \( p(t) = 0.296t^2 - 0.229t - 0.013 \) hundred patents gives the number of social network software patents that are published (and publicly available), \( t \) years after 2003, \( 0 \leq t \leq 7 \).
   Checkpoint: \( p(2) = 0.713 \)
   
a. Find \( p(5) = \) ________________ (correct to three decimal places).

   Write a sentence of interpretation.

b. According to the model, in what year will there be 15 hundred social network software patents published?

   Answer correctly to three decimal places before giving the year. \( t = \) __________ , in the year__________

   (_______ / 7 pts)
2. The following data gives the population of Lebanon, in millions of people, for various years.

<table>
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</tr>
</thead>
<tbody>
<tr>
<td>Population, in million people</td>
<td>3.51</td>
<td>3.90</td>
<td>4.19</td>
<td>4.37</td>
<td>4.51</td>
</tr>
</tbody>
</table>

a. How many concavities does the scatterplot indicate?  
   Circle one: zero one two

b. Align the input values to 2000.

c. Find a logarithmic function to model the aligned data. Write a completely defined logarithmic model.

d. To find a new model for the data in which the function gives the year (as output) in terms of the population of Lebanon (as input), what type of function would be most appropriate to use?
   Circle One: Linear Exponential Logarithmic Logistic Quadratic Cubic  

   (_______ /11 pts)

3. An airline offers a flight between New York and Los Angeles. Its base operating costs are $20,000 per flight. Each additional passenger increases the costs by $75 because of additional fuel, service, etc. The airline sells tickets for the flight at a price of $392 per ticket,

a. Find an equation, \( C(x) \), for total cost, in dollars, of the flight when there are \( x \) passengers.

   \[ C(x) = \] _________

b. Find an equation, \( R(x) \), for revenue, in dollars, from the flight when there are \( x \) passengers.

   \[ R(x) = \] _________

c. How many passengers must the plane carry in order to first break even on the flight?

   _________ passengers  

   (_______ / 6 pts)
4. When Sally received an inheritance, she invested a sum of money in a certificate of deposit paying 0.5% compounded monthly. After 6 years, with no further deposits or withdrawals, the balance was $32,509.97. What was Sally’s initial investment? For full credit, state any formulas or equations used in finding the answer. Include units with the answer.

\[
\text{Initial Investment} = \frac{\text{Final Balance}}{(1 + \text{Rate})^{\text{Number of Periods}}} \times (100 - \text{Rate})
\]

\[
\text{Initial Investment} = \frac{32509.97}{(1 + 0.005)^{6 \times 12}} \times (100 - 0.5)
\]

\[
\text{Initial Investment} \approx 30,000
\]

5. Label the two graphs (fill in the boxes) with the correct functions: \( f(x) = 1 + 2 \ln x \) and \( g(x) = 3 - 2 \ln x \)

6. \( f(x) \) homicides gives the number of workplace homicides in the US, \( x \) years after 2005, \( 0 \leq x \leq 3 \). \( f(x) \) is graphed below. Mark an “X” directly on the function graph (not the axes) to indicate the location of the inflection point.

7. A correctly bubbled scantron with a #2 pencil, a correct XID, a correctly bubbled XID, a correctly bubbled test version, AND a signed academic integrity statement (on the front of the test) earns 1 point.

\text{END OF TEST}