Printed Name: ______________________________  Section #: _______ Instructor:____________________

Please do not ask questions during this exam. If you consider a question to be ambiguous, state your assumptions in the margin and do the best you can to provide the correct answer. See below and page two for formulas, general directions, and calculator troubleshooting tips.

Any communication with any person (other than the instructor or the designated proctor) during this exam in any form, including written, signed, verbal or digital, is understood to be a violation of academic integrity.

All devices, such as computers, cell phones, cameras and PDAs must be turned off while the student is in the testing room.

The only calculators to be used are TI-83, TI-83+, TI-84 or TI-84+. You may NOT borrow or share a calculator with another person taking this test.

Statement of Academic Integrity: I have not and will not give or receive improper aid on this test.

In signing below, I acknowledge that I have read, understand, and agree to these testing conditions.

Student’s Signature: _____________________________________________________________________

(This test will not be accepted for grading unless it bears the signature of the student.)

<table>
<thead>
<tr>
<th></th>
<th>FR#1</th>
<th>FR #2</th>
<th>FR #3</th>
<th>FR #4</th>
<th>FR #5</th>
<th>FR #6</th>
<th>scantron</th>
<th>Free Response Total</th>
<th>Multiple Choice Total</th>
<th>Total</th>
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</thead>
<tbody>
<tr>
<td>Possible points</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td>10</td>
<td>2</td>
<td>9</td>
<td>1</td>
<td>41</td>
<td>59</td>
<td>100</td>
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<td>Points Earned</td>
<td></td>
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</table>

Useful Formulas: \[ f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \]; \[ f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} \]
General Directions:
Show work where possible. Answers without supporting work (where work is appropriate) may receive little credit.

Do not round intermediate calculations.

Answers in context ALWAYS require units.

Round your answers to 3 decimal places UNLESS the answer needs to be rounded differently to make sense in the context of the problem OR the directions specify another type rounding OR the complete answer has less than 3 decimal places.

When you are asked to write a model, include all components of a model: an equation, a description of the input including units, a description of the output including units, and the interval when known.

When asked to write a sentence of practical interpretation, answer the questions: when?, what?, and how much? using ordinary, conversational language. DO NOT use math words, terms, or unnecessary phrases.

Always use a ruler when estimating values off of a graph.

**HINTS FOR TROUBLESHOOTING YOUR CALCULATOR:**

- If you lose your L1, L2, etc., you may reinsert them using STAT 5 (set-up editor) enter.

- The SCATTER PLOT will not show unless Plot 1 has been turned on and there is data in L1 and L2.

- ZOOM 0 may not work for graphing if Plot 1 is turned on.

- DIM MISMATCH error usually means that the lists in L1 and L2 are not of equal length.

- DATA TYPE error usually means that you already have something in Y1 and you need to clear it before you can paste a new equation.

- INVALID DIM error usually means that your plot(s) are on, but that you have no data in the lists. Refer to the second hint above.

- If your batteries die, raise your hand and hold up your calculator. If your instructor has an extra calculator available, he/she will loan it to you for a few minutes.

- SYNTAX ERROR: Try GO TO. This will happen if you use a subtraction minus sign when you should use a negative sign.
MULTIPLE CHOICE: 59 points (3 points each unless otherwise noted)

D B C C A | B C B A C | C D B D D | B A D A A | B

Use a #2 pencil and completely fill each bubble on your scantron to answer each multiple choice question. (For future reference, circle your answers on this test paper.) There is no penalty for guessing on multiple choice. If you indicate more than one answer, or you leave a blank, the question will be marked as incorrect.

Use the following to answer the next two questions:

\[ u(t) = 43.05 \times (1.08^t) \]  

million subscribers gives the number of Cable subscribers in the US, where \( t \) is the number of years since 2002, \( 0 \leq t \leq 8 \).  

Checkpoint: \( u(2) = 50.21352 \)

1. Complete the following sentence:

Between 2002 and 2010, the number of Cable subscribers in the US increased by \( \boxed{85.093} \) percent.

a. 4.579  
   b. 36.633  
   c. 45.973  
   d. 85.093

2. Complete the following sentence:

In 2005, the number of Cable subscribers in the US was increasing by \( \boxed{7.696} \) percent per year.

a. 4.174  
   b. 7.696  
   c. 12.994  
   d. 54.231

3. In 2012, the number of Facebook users, in millions, was 750. Three years later, the number of Facebook users jumped to 1500 million users.

Complete the following sentence:

Between 2012 and 2015, the number of Facebook users increased on average by \( \boxed{250} \) million users per year.

a. 0.004  
   b. 100  
   c. 250  
   d. 750

4. Suppose that a function \( f(x) \) has an average rate of change of 2.2 between \( x = 1 \) and \( x = 4 \).

Which one of the following statements is NOT always true?  

\[ \frac{df}{dx} \bigg|_{x=4} - \frac{df}{dx} \bigg|_{x=1} = 2.2 \]

a. \( f(4) - f(1) = 6.6 \)  
   b. 2.2 is the slope of the secant line between \( x = 1 \) and \( x = 4 \)  
   c. \( \frac{df}{dx} \bigg|_{x=4} - \frac{df}{dx} \bigg|_{x=1} = 2.2 \)  
   d. \( \frac{f(4) - f(1)}{4-1} = 2.2 \)
5. Consider the following graph of a function $f$. Which ONE of the following statements is TRUE?  

\[ \lim_{x \to -1} f(x) \text{ exists, even though } f \text{ is not continuous at } x = -1. \]

\[ f'(1) \text{ exists.} \]

\[ \text{The function } f \text{ has a positive slope at } x = 4. \]

\[ \text{The derivative of the function } f \text{ exists at } x = 3. \]

6. Based on data from 1940 to 2000, the average monthly Social Security benefit for men may be modeled by $P(t)$ dollars, where $t$ is the number of years since 1940.

Write a sentence of interpretation for $P'(20) = 8.455$.

a. In 1960, the average monthly social security benefit for men was 8.455 dollars.

b. In 1960, the average monthly social security benefit for men was increasing by 8.455 dollars per year.

c. In 1960, the average monthly social security benefit for men was increasing by 8.455 percent per year.

d. Between 1940 and 1960, the average monthly social security benefit for men increased on average by 8.455 dollars per year.

7. If a continuous and smooth function $f(x)$ has a relative minimum at $x = 2$, its slope graph $f'(x)$ will have a \underline{\hspace{5cm}} at $x = 2$.

a. discontinuity  
b. relative minimum  
c. zero  
d. negative slope
Consider the graph of the function \( f(x) \) on the interval \([-2,10]\) to answer the next three questions:

8. On the interval \((-2, 10)\), at which input value(s) is the above function continuous but not differentiable? (Recall that “differentiable” means that the derivative exists.) [2 pts]
   a. \( x = 0 \) and \( x = 2 \)   b. \( x = 3 \) only   c. \( x = 3 \) and \( x = 7 \)   d. \( x = 7 \) only

9. On the interval \((-2, 10)\), \( \frac{df}{dx} > 0 \) on what interval(s)?
   a. \(-2 < x < 0 \) and \( 2 < x < 3 \)   b. \( 0 < x < 2 \) and \( 3 < x < 7 \) and \( 7 < x < 10 \)
   c. \(-1.25 < x < 10 \)   d. \( 1 < x < 3 \)

10. At \( x = 1 \), the slope graph \( f'(x) \) will have a(n) ___________.
    a. inflection point   b. relative maximum   c. relative minimum   d. zero
11. Which ONE of the following graphs is the slope graph of the function graphed \( f(x) \) to the right?

![Graphs a, b, c, d]

\[ f(x) \]

12. Find the derivative of \( r(x) = 5e^x - \pi^3 \). \( r'(x) = \) ___________.

a. \( 5xe^{x-1} - 3\pi^2 \)  
   b. \( 5e^x - 3\pi^2 \)  
   c. \( 5xe^{-x-1} \)  
   d. \( 5e^x \)

13. Given \( g(x) = \frac{3}{x^{-\frac{3}{2}}} \), \( g'(x) = \) ___________.

a. \( 3x^{-\frac{7}{2}} \)  
   b. \( -2x^{-\frac{5}{2}} \)  
   c. \( -2x^{\frac{1}{2}} \)  
   d. \( \frac{3}{2}x^{-\frac{5}{2}} \)

14. Given \( p(x) = \frac{3x^3 + 8x^2 - 4x + 1}{4x} \), \( p'(x) = \) ___________.

a. \( \frac{3}{4}x^2 + 2x - 1 + \frac{1}{4}x^{-1} \)  
   b. \( \frac{3}{2}x + 2 - 4x^{-2} \)  
   c. \( \frac{9x^2 + 16x - 4}{4} \)  
   d. \( \frac{3}{2}x + 2 - \frac{1}{4}x^{-2} \)

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Use the graph below to answer the next four questions:

15. Consider the points A, B, and C on the graph with input values \( x = -2, x = -1, \) and \( x = 3, \) respectively. List the points in order from the LEAST to GREATEST slope.
   [2 pts]
   a. B, A, C  
   b. A, B, C  
   c. C, A, B  
   d. C, B, A

16. Draw a tangent line to \( f(x) \) at the inflection point \( C(3,3) \).
   On the interval \( 1 < x < 3, \) the tangent line to \( f(x) \) at the inflection point \( (3,3) \) lies _________ the graph and on the interval \( 3 < x < 5, \) the tangent line to \( f(x) \) at the inflection point \( (3,3) \) lies _________ the graph.
   a. below; above  
   b. above; below  
   c. below; below  
   d. above; above

17. \( f'(3) \) is approximately equal to ______________.
   a. -2.5  
   b. -0.4  
   c. -1  
   d. undefined

18. Which ONE of the following graphs is the slope graph of the function \( f(x) \) graphed above?
   a.  
   b.  
   c.  
   d. 
19. If \( f(p) = \ln p \) and \( p(x) = 4x^2 - 5x + 1 \), then the derivative of the composition \( f(p(x)) \) is _________.

\[ \frac{8x - 5}{4x^2 - 5x + 1} \quad \text{a.} \]

b. \( \frac{1}{x} \cdot (8x - 5) \)

c. \( \frac{1}{4x^2 - 5x + 1} \)

d. \( \frac{1}{8x - 5} \)

Use the following to answer the next two questions:

\( n(x) \) hundred customers gives the number of customers shopping at a car dealership monthly, after \( x \) hours of a television advertising campaign has aired.

\( a(n) \) is the number of automobiles sold monthly when \( n \) hundred customers shop at the car dealership monthly.

After 4 hours of television advertising, the number of monthly customers at a car dealership is 3 hundred, and this number is increasing by 0.2 hundred customers per hour of television advertising.

When there are 3 hundred monthly customers at a car dealership, 15 automobiles are sold monthly, and this number is increasing by 3.25 automobiles per hundred customers.

Let \( f(x) = a(n(x)) \).

20. Find the derivative of \( f(x) \) to answer the following question.

How quickly are monthly sales of automobiles changing, with respect to advertising time, after 4 hours of a television advertising campaign has aired?

\[ \text{a.} \ 0.65 \quad \text{b.} \ 0.80 \quad \text{c.} \ 3 \quad \text{d.} \ 9.75 \]

21. What are the units for the answer to the previous question?

\[ \text{a.} \ \text{automobiles per month} \quad \text{b.} \ \text{automobiles per hour} \]

c. \( \text{hundred customers per hour} \)

d. \( \text{automobiles per hundred customers} \)
FREE RESPONSE: 41 points

Show work where possible. Read the directions on rounding, inclusion of units, and writing models and sentences at the front of the test.

1. Find the derivative for each of the given functions. Simplify exponents and coefficients, and use exact numbers. For full credit, include proper use of derivative notation and equal signs. (_______ / 6 pts)

   a. \( f(x) = 8\sqrt{x} + 3x^{-2} \)
      \[ f'(x) = 4x^{-\frac{1}{2}} - 6x^{-3} \]
      a. 2.5 pts
      b. 3.5 pts

   b. \( f(x) = 3(2.5^x) - x + 5\ln x \)
      \[ f'(x) = 3(\ln 2.5)(2.5^x) - 1 + \frac{5}{x} \]

2. The table gives the price of a ticket to a certain NBA team and the corresponding monthly profit for that team.

<table>
<thead>
<tr>
<th>Ticket price, in dollars</th>
<th>40</th>
<th>45</th>
<th>50</th>
<th>55</th>
<th>60</th>
<th>65</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly profit, in thousand dollars</td>
<td>3250</td>
<td>3480</td>
<td>3670</td>
<td>3900</td>
<td>3720</td>
<td>3520</td>
</tr>
</tbody>
</table>

   a. Calculate the change in profit when the ticket price rises from $60 to $65. Show work and include units with the answer.
      \[ 3520 - 3720 = -200 \text{ thousand dollars} \]

   b. Calculate the average rate of change of profit when the ticket price rises from $60 to $65. Show work and write a sentence of interpretation.
      \[ \frac{3520 - 3720}{65 - 60} = -40 \text{ thousand dollars per dollar (in ticket price)} \]

      When the ticket price rises from 60 to 65 dollars, the monthly profit for a certain NBA team decreases on average by 40 thousand dollars per dollar (in ticket price).

      (_______ / 7 pts)
3. \( f(x) = -3.089x^2 + 44.606x - 91.061 \) thousand dollars gives the average monthly revenue for an independent bookstore, \( x \) years after 2000, \( 4 \leq x \leq 11 \).

**Checkpoint:** \( f(2) = -14.205 \)

Use the numeric method to determine the instantaneous rate of change in the bookstore’s average monthly revenue in 2010.

a. Fill in the table below with entries rounded correctly to THREE decimal places.

(Answers will be marked wrong if not rounded to three decimal places.)

<table>
<thead>
<tr>
<th>( x \to 10^- )</th>
<th>( \frac{f(x) - f(10)}{x - 10} )</th>
<th>( x \to 10^+ )</th>
<th>( \frac{f(x) - f(10)}{x - 10} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.99</td>
<td>-17.143</td>
<td>10.01</td>
<td>-17.205</td>
</tr>
<tr>
<td>9.999</td>
<td>-17.171</td>
<td>10.001</td>
<td>-17.177</td>
</tr>
<tr>
<td>9.9999</td>
<td>-17.174</td>
<td>10.0001</td>
<td>-17.174</td>
</tr>
<tr>
<td>9.99999</td>
<td>-17.174</td>
<td>10.00001</td>
<td>-17.174</td>
</tr>
</tbody>
</table>

b. Using the numeric method, what is the instantaneous rate of change in the bookstore’s average monthly revenue in 2010? Answer correct to three decimal places and include units.

\[-17.174 \text{ thousand dollars per year} \]

(_______ /6 pts)

4. The average monthly Social Security benefit for men may be modeled by \( P(t) = 0.349t^2 - 5.505t + 35.130 \) dollars, where \( t \) is the number of years since 1940, \( 0 \leq t \leq 60 \).

**Checkpoint:** \( P(2) = 25.516 \)

a. Write a completely defined *rate-of-change model* for \( P(t) \) by filling in the blanks:

\[ P'(t) = ____ 0.698t - 5.505 ____ ____ dollars per year ____ ____

(equation) (units)

gives _the rate of change in the average monthly Social Security benefit for men, where \( t \) is the number of years since 1940, \( 0 \leq t \leq 60 \).

(a) 5 pts:
1 pt equation; 1 pt units; 1 pt "rate of change"; 1 pt "output description"

b. Find \( \frac{dP}{dt} \bigg|_{t=50} = ____ 29.395 ____ ____ \) (correct to three decimal places) and write the answer in a sentence of interpretation.

In 1990, the average monthly Social Security benefit for men was increasing by 29.395 dollars per year.

(_______ /10 pts)
5. Fill in the blank of each of the following sentences with one of the following:

- **Average rate of change (AROC)**
- **Instantaneous rate of change (IROC)**

The formula \( \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \) finds **IROC** for a function \( f(x) \), whereas \( \frac{f(x+h) - f(x)}{h} \) finds **AROC** between points on the graph of a function \( f(x) \). (1 pt per blank)

6. Use the limit definition of the derivative to find the derivative of \( f(x) = 8x^2 - 2.5x + 3 \).

(Complete your answer using simple rate of change formulas.) For full credit, continue from the general limit definition (provided below), clearly showing all necessary algebraic steps (cancellations, expansions, etc.) and including proper use of notation and equal signs.

\[
\frac{d}{dx} f(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}
\]

\[
= \lim_{h \to 0} \frac{[8(x+h)^2 - 2.5(x+h) + 3] - [8x^2 - 2.5x + 3]}{h}
\]

\[
= \lim_{h \to 0} \frac{[8x^2 + 16xh + 8h^2 - 2.5x - 2.5h + 3 - 8x^2 + 2.5x - 3]}{h}
\]

\[
= \lim_{h \to 0} \frac{16xh + 8h^2 - 2.5h}{h} = \lim_{h \to 0} \frac{h(16x + 8h - 2.5)}{h}
\]

\[
= \lim_{h \to 0} (16x + 8h - 2.5) = 16x - 2.5
\]

Thus, \( f'(x) = 16x - 2.5 \)

- **3 pts:** Find slope of secant using given function: \([f(x+h) - f(x)]/h\)
- **1 pt:** Square \((x+h)\) correctly
- **2 pts:** Distribute \(8\), \(-2.5\), and \(-1\) (minus sign) correctly
- **1 pt:** Combine like terms and show the result.
- **1 pt:** Show the limit of a completely simplified expression
- **1 pt:** Evaluate limit of simplified expression to find derivative.

Deductions: if written limit notation is not written correctly throughout proof and/or if equal signs not in correct places and

(_________ / 9 pts)

7. 1 point for correctly filling out and bubbling the scantron with a #2 pencil, a correct XID, a correct test version, AND the front of the test is completed with your signature on the academic integrity statement.

**END OF TEST**