Instructions: You are not permitted to use a calculator on any portion of this test. You are not allowed to use any textbook, notes, cell phone, laptop, PDA, or any technology on either portion of this test. All devices must be turned off while you are in the testing room.

During this test, any communication with any person (other than the instructor or a designated proctor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.

No part of this test may be removed from the examination room.

Read each question very carefully. In order to receive full credit for the free response portion of the test, you must:
1. Show legible, logical, and relevant justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.
4. Give exact values whenever possible.

You have 90 minutes to complete the entire test.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student’s Signature: ____________________________________________
Read each question carefully. In order to receive full credit you must show legible, logical, and relevant justification which supports your final answer. Give answers as exact values. You are NOT permitted to use a calculator on any portion of this test.

1. (8 pts. each) Find the indicated derivatives. Assume \( g(x) \) is a differentiable function.

   a) Find \( f'(x) \) if \( f(x) = \pi^3 g(\sec(7^x)) \).

   b) Find \( \frac{dy}{dx} \) if \( \tan(x + 2y) = e^{y^3} \).
c) Find $f'(x)$ if $f(x) = \ln \left( \frac{3'}{\sin(5x)} \right)$. Simplify your answer.

d) Find $\frac{dy}{dx}$ if $y = x^{\sin^{-1}x}$ (also written $y = x^{\arcsin x}$)
2. (12 pts.) A modern art exhibit at a museum contains a steel sphere with a radius of 8 meters.

a) (7 pts.) Heating causes the sphere to expand. Use a differential to estimate the change in the surface area of the sphere if heat causes the radius to increase from 8 meters to 8.1 meters.

Note: The surface area $S$ of a sphere of radius $r$ is $S = 4\pi r^2$.

b) (5 pts.) Assume the paint on the surface of the sphere will crack if the volume of the sphere increases by 64 cubic meters. Use the differential to estimate the increase in radius that would cause the paint to begin to crack.

Note: The volume $V$ of a sphere of radius $r$ is $V = \frac{4\pi}{3} r^3$. 
3. (12 pts.) A tank in the shape of an inverted cone has a radius of 4 meters and a height of 5 meters. It drains into a cylindrical tank with a radius of 4 meters and a height of 6 meters (see figure).

![Figure of a conical and cylindrical tank](image)

a) (8 pts.) Consider the conical tank. When the height of water in the conical tank is exactly one meter, the height of the water is decreasing at 0.5 meters per hour. At what rate is water flowing from the conical tank at this moment?

Note: The volume $V$ of a cone is $V = \frac{\pi}{3} r^2 h$

b) (4 pts.) Consider the cylindrical tank. At what rate is the water level in the cylindrical tank changing when the water level in the conical tank is exactly one meter? (the same moment from part (a))

Note: The volume $V$ of a cylinder is $V = \pi r^2 h$
4. (15 pts.) Let \( f \) and \( g \) be differentiable functions. Use the values in the table below to answer the questions that follow.

<table>
<thead>
<tr>
<th></th>
<th>( x=0 )</th>
<th>( x=1 )</th>
<th>( x=2 )</th>
<th>( x=3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f(x) )</td>
<td>0</td>
<td>1</td>
<td>8</td>
<td>-27</td>
</tr>
<tr>
<td>( f'(x) )</td>
<td>0</td>
<td>3</td>
<td>12</td>
<td>27</td>
</tr>
<tr>
<td>( g(x) )</td>
<td>1</td>
<td>3</td>
<td>-9</td>
<td>19</td>
</tr>
<tr>
<td>( g'(x) )</td>
<td>4</td>
<td>-2</td>
<td>4</td>
<td>12</td>
</tr>
</tbody>
</table>

a) (7 pts.) Let \( h(x) = f(x)e^{g(x)} \). Is \( h \) increasing or decreasing at \( x = 1 \)? Show work that supports your answer.

b) (8 pts.) Let \( h(x) = \tan^{-1}(g(x)) \). Find the equation of the line tangent to \( h(x) \) at \( x = 0 \). Put your final answer in slope-intercept form (\( y = mx + b \)).
5. (12 pts.) The exponential growth and decay function \( y = y_0 e^{kt} \) can be used to model the amount of a drug in the bloodstream of a patient \( t \) hours after being administered.

a) (4 pts.) The half-life of morphine is 3 hours. Assuming an initial dose of 40 mg, find the exponential function that models \( y \), the amount of morphine in the blood \( t \) hours after being administered.

b) (4 pts.) How long will it take for an initial dose of 40 mg of morphine to decay to 13 mg? Give your final answer in terms of natural logarithms.

c) (4 pts.) At what rate is the amount of morphine exiting the blood 6 hours after the administration of a 40 mg dose? Simplify your final answer to the form \( a \ln(1/2) \), where \( a \) is a constant.
6. (10 pts.) Find the absolute maximum and absolute minimum values of the function on the given interval and the $x$-values where they occur. Put your final answers in the blanks below, with supporting work below the answers.

$$f(x) = x^{2/3}(2x - 5), \quad x \in \left[-1, \frac{5}{2}\right]$$

Answers: Absolute \textbf{minimum} of \underline{_______________} at $x = \underline{______________}$.

Absolute \textbf{maximum} of \underline{_______________} at $x = \underline{______________}$. 
7. (7 pts.) Prove the following identity.

\[
\sinh(2x) = 2 \sinh x \cosh x
\]