PLEASE READ DIRECTIONS!!!!

In order to receive full credit for the free response portion of the exam, you must:
1. Show legible and logical (relevant) justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.

You have 1 hour 30 minutes to complete the entire exam.

On my honor, I have neither given nor received inappropriate or unauthorized information during this exam.

Student’s Signature: ____________________________________________

Do not write below this line.

<table>
<thead>
<tr>
<th>Free Response Problem</th>
<th>Possible Points</th>
<th>Points Earned</th>
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<td>Free Response</td>
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<td>Multiple Choice</td>
<td>48</td>
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<td>Test Total</td>
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Multiple Choice: (Questions 1 - 16) Answer the following questions on the scantron provided using a #2 pencil. Bubble the response that best answers the question. Each multiple choice correct response is worth 3 points. For your record, also circle your choice on your exam since the scantron will not be returned to you. Only the responses recorded on your scantron will be graded.

1. Which geometric shape is used to represent areas for a uniform distribution?
   A) Circle
   B) Rectangle
   C) Bell curve
   D) Triangle

2. A professor receives, on average, 24.7 e-mails from students the day before the midterm exam. To compute the probability of receiving at least 10 e-mails on such a day, he will use what type of probability distribution?
   A) Poisson distribution
   B) Binomial Distribution
   C) Normal Distribution
   D) Uniform Distribution

3. What is the z-score for the first quartile?
   A) -0.67
   B) 0.25
   C) 0.60
   D) -0.25
   E) Cannot be determined from the information given.
4. The number of hours a college student sleeps the night before finals week begins has a distribution that is quite skewed. Let \( \bar{X} \) represent the mean hours slept the night before finals week for a random sample of 45 college students. What can be said about the sampling distribution of \( \bar{X} \)?

A) The sampling distribution of \( \bar{X} \) may not be normally distributed because the number of hours a college student sleeps the night before finals week begins is not normally distributed.

B) The sampling distribution is \( \bar{X} \) is approximately normally distributed because hours of sleep is a continuous random variable.

C) The sampling distribution of \( \bar{X} \) is approximately normally distributed because the sample size is large enough.

D) The sampling distribution of \( \bar{X} \) may not be normally distributed because the sample size is not large enough.

5. The probabilities and payoffs (X) for betting $5 on the number 7 in roulette are summarized below. For repeated games, what is the standard deviation of the payoffs in the long run? (Note: you do not need to know how to play roulette in order to solve this problem)

<table>
<thead>
<tr>
<th>Event</th>
<th>X</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lose</td>
<td>-$5</td>
<td>0.9737</td>
</tr>
<tr>
<td>Win (net gain)</td>
<td>$175</td>
<td>0.0263</td>
</tr>
</tbody>
</table>

A) -0.27
B) 28.87
C) -19.67
D) 5.37

6. For which of the following will the sample proportion tend to differ least from sample to sample?

(A) Random samples of size 60 from a population with \( p = 0.1 \)
(B) Random samples of size 50 from a population with \( p = 0.1 \)
(C) Random samples of size 40 from a population with \( p = 0.5 \)
(D) Random samples of size 50 from a population with \( p = 0.6 \)
(E) Random samples of size 60 from a population with \( p = 0.6 \)
7. The histograms shown here are approximate sampling distributions of the sample mean. Each histogram is based on selecting 1000 different samples, each of size $n$. All three histograms were constructed by sample from the same population, but the sample sizes were different. Which histogram was based on samples with the largest sample size, $n$?

(A) Graph I
(B) Graph II
(C) Graph III
(D) All three have the same sample size
(E) It is not possible to tell from the histogram
8. True or False: If a random variable is discrete, it means that the outcome for the random variable can take on only one of two possible values.

A) True  
B) False

9. The life of batteries (X) is distributed normally. The standard deviation of the lifetime is 15 hours and the mean lifetime of a battery is 450 hours. Find the probability of a battery lasting for at most 477 hours. Give the appropriate probability statement.

A) \( P(X \leq 477) \)  
B) \( P(X < 477) \)  
C) \( P(Z < 1.80) \)  
D) All of the above  
E) B and C only

10. A certain insecticide kills 70% of all insects in laboratory experiments. A sample of 5 insects is exposed to the insecticide in a particular experiment. Let \( X \) be the number of insects that survive. Below is the probability distribution of \( X \).

<table>
<thead>
<tr>
<th>( X )</th>
<th>( P(X) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00243</td>
</tr>
<tr>
<td>1</td>
<td>0.02835</td>
</tr>
<tr>
<td>2</td>
<td>0.13230</td>
</tr>
<tr>
<td>3</td>
<td>0.30870</td>
</tr>
<tr>
<td>4</td>
<td>0.36015</td>
</tr>
<tr>
<td>5</td>
<td>0.16807</td>
</tr>
</tbody>
</table>

If this experiment were to be repeated many times, what would be the average number of insects that survive in the long run?

A) 4 insects  
B) 3.5 insects  
C) 1 insect  
D) 5 insects
11. Is the following random variable discrete or continuous?

   Y represents the length of time it takes an individual to finish a 5k race

   A) Continuous  
   B) Discrete  
   C) Neither

12. Over the past 100 years, the mean number of annual major earthquakes in the world is 0.93. Assuming that the Poisson distribution is a suitable model, you would like to find the probability that the number of earthquakes in a randomly selected year is 3. What is the value of lambda (λ) for the given situation?

   A) 100  
   B) 0.93  
   C) 3  
   D) 0.05

13. You are to roll two standard 6 sided dice (one red and one blue). Let the random variable X represent the difference between the rolls of the two dice (red – blue). What are the possible values of X?

   A) \{1, 2, 3, 4, 5, 6\}  
   B) \{0, 1, 2, 3, 4, 5\}  
   C) \{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}  
   D) \{0, 1, 2\}

14. The local police department must write, on average, 5 tickets a day to keep departments revenues at budgeted levels. Suppose the number of tickets written per day follows a Poisson distribution with a mean of 6.5 tickets per day. Interpret the value of the mean.

   A) Half of the days have less than 6.5 tickets written and half of the days have more than 6.5 tickets written
   B) The mean has no interpretation since 0.5 ticket can never be written
   C) If we sampled all days, the arithmetic average or expected number of tickets written would be 6.5 tickets per day.
   D) The number of tickets that is written most of is 6.5 tickets per day
15. Find the area under the standard normal curve to the left of \( z = 0.89 \).

A) 1.23  
B) 0.8133  
C) 0.2685  
D) 0.3133  

16. Thirty percent of all automobiles undergoing an emissions inspection at a certain inspection station fail the inspection. Let the binomial random variable \( X \) be the number of cars out of 12 that fail the inspection. How many different ways can 10 cars fail out of 12?

A) 132  
B) 66  
C) 10  
D) Approximately 0
1. (5 pts) A breeder of show dogs is interested in the number of female puppies in a litter. If a birth is equally likely to result in a male or a female puppy, give the probability distribution of the variable $X =$ number of female puppies in a litter of size 4. **Fill in the table below with probabilities to 4 decimal places or REDUCED fractions, be sure to show your work. Remember no credit will be given for values with no work shown.**

<table>
<thead>
<tr>
<th>$x$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X = x)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</table>
2. (5pts) Mars, Inc. claims that 24% of its M&M plain candies are blue. A random sample of 100 M&M’s is chosen. What is the probability that exactly 28 are blue? **Fill in the blanks in the equation below.**

\[
\text{(Probability Statement)} = \frac{\text{(Formula with values filled in)}}{\text{(Value to 4 decimal places)}}
\]

3. (5pts) Consider the variable \(X =\) time required for a college student to compete a standardized exam. Suppose that for the population of students at a particular university, the distribution of \(X\) is well approximated by a normal curve with mean 50 minutes and standard deviation 5 minutes. How much time should be allowed for the exam if we wanted 95% of the students taking the test to be able to finish in the allotted time? **Show work, give value to 2 decimal places (with units).**
4. Assume that the number of network errors experienced in a day on a local area network (LAN) is distributed as a Poisson random variable. The mean number of network errors experienced in a day is 3.1.

   a. **(5 pts)** What is the probability that in any given day fewer than 3 network errors will occur? Define your random variable, give the appropriate probability statement, show work indicating which values of your random variable are included, and give the value to 4 decimal places.

   b. **(5 pts)** What is the probability that 2 errors will occur in 3 days? Define your random variable, give the appropriate probability statement, show work by filling in values to an equation, and give the value to 4 decimal places.
5. (5 pts) A certain chemical process reaction time (X) has a Uniform Distribution from 1 minute to 7 minutes. What is the probability that the process will react after 5 minutes? **Give the appropriate probability statement, draw the appropriate picture for the situation, show work, and give value to 4 decimal places.**

6. (6pts) For a Standard Normal random variable (Z), find the probability that Z is in between -0.82 and 2.23. **Give the appropriate probability statement, draw the appropriate picture for the situation, show work, and give value to 4 decimal places.**
7.  (5pts) Sophie is a dog that loves to play catch. Unfortunately, she isn’t very good, and the probability that she catches a ball is only 0.1. Let X be the number of tosses required until Sophie catches a ball. Sophie’s owner can throw the ball no more than 50 times.

Does X have a binomial distribution? (Circle one)

Yes, Binomial  No, Not Binomial

If you answered Binomial, what are the values of the binomial parameters n and p? If you answered not Binomial, which criteria for binomial is violated?
8. Let $X$ denote the time (in minutes) that it takes a fifth-grade student to read a passage. Suppose that $X$ is normally distributed with mean of 2 minutes and standard deviation of 0.8 minutes.

a. **(5pts)** What is the probability that a randomly selected fifth grader takes longer than 4 minutes to read the passage? **Show work and fill in the blanks below. Remember values with no work shown will receive no credit.**

\[
\text{(Probability Statement)} = \frac{X - \mu}{\sigma} = \frac{4 - 2}{0.8} = \frac{2}{0.8} = 2.5
\]

b. **(5pts)** If we randomly sample 9 fifth grade students, what is the probability that the sample mean amount of time it takes to read a passage is less than 1.5 minutes? **Show work and fill in the blanks below. Remember values with no work shown will receive no credit.**

\[
\text{(Probability Statement)} = \frac{\bar{X} - \mu}{\sigma / \sqrt{n}} = \frac{1.5 - 2}{0.8 / \sqrt{9}} = \frac{-0.5}{0.8 / 3} = \frac{-0.5}{0.2667} = -1.86
\]

9. **(1 pt)** If your scantron is correctly bubbled with a #2 pencil, with your correct XID (with the “C” bubbled in as a “0”), your correct test version, AND the front of your test is completed with your signature on the academic integrity statement, you earn 1 point.

END OF TEST