Multiple Choice: Use a #2 pencil and completely fill in each bubble on your scantron to indicate the answer to each question. Each question has one correct answer. If you indicate more than one answer, or leave a blank, the question will be marked as incorrect. In this section there are 16 multiple choice questions. Each question is worth 3 points for a total of 48 points. For future reference, circle your answers on this test paper as you will not receive your Scantron back with your test.

A sample of an isotope $^{18}\text{Ar}^{39}$ will decay at a rate $r(t) = -0.078(0.997^t)$ grams per year, where $t$ is the number of years since the sample began to decay. [Check: $r(2) \approx -0.0775$]

1. According to the model, how much of the $^{18}\text{Ar}^{39}$ will eventually decay?
   a. 26.105 grams  
   b. 25.961 grams  
   c. 24.674 grams  
   d. 20.181 grams

When asked to algebraically evaluate $\int_{3}^{\infty} 50e^{10x} \, dx$, a student has reached the following step in his work. (The work in between has been intentionally omitted.)

$$\int_{3}^{\infty} 50e^{10x} \, dx = \lim_{N \to \infty} 5e^{10N} - \lim_{N \to \infty} 5e^{10(3)}$$

2. Which of the following is the final result of the algebraic process for evaluating $\int_{3}^{\infty} 50e^{10x} \, dx$?
   a. $\int_{3}^{\infty} 50e^{10x} \, dx = -5e^{30}$  
   b. $\int_{3}^{\infty} 50e^{10x} \, dx = -\infty$  
   c. $\int_{3}^{\infty} 50e^{10x} \, dx = \infty$  
   d. $\int_{3}^{\infty} 50e^{10x} \, dx = -5$
A major entertainment complex has determined that they will invest \( R(t) = 17.2 + 1.3t \) million dollars per year \( t \) years from now, where \( 0 \leq t \leq 5 \). Assume a continuous income stream and an APR of 4%, compounded continuously. [Check: \( R(2) = 19.8 \)]

Use this information to answer the next two questions.

3. Find the 5-year future value of the investment.
   a. 105.848 million dollars
   b. 93.165 million dollars
c. 112.593 million dollars
d. 102.250 million dollars

4. Find the 5-year present value of their investment.
   a. 83.715 million dollars
   b. 137.521 million dollars
c. 102.250 million dollars
d. 92.183 million dollars

5. Having received a large inheritance, a child’s parents wish to establish a trust for the child’s college education. If 7 years from now they need 50 thousand dollars, how much should they set aside in a trust now, if they invest into an account which pays 5.2% interest compounded continuously?
   a. 34.745 thousand dollars
   b. 31.042 thousand dollars
c. 71.954 thousand dollars
d. 37.143 thousand dollars

6. The owners of a small company posted profits of 420 thousand dollars last year. The company expects their profits to remain the same over the next 5 years and they plan on investing all of their profits into an account where interest is compounded continuously at 3.5%. Assuming a continuous income stream, how much interest will their investment earn over the next five years?
   a. 194.955 thousand dollars
   b. 368.439 thousand dollars
c. 894.245 thousand dollars
d. 320.131 thousand dollars
\[ W(x) = 0.05x^2 - 0.4x + 10.8 \] hundred gallons represents the amount of water available to a farmer for irrigation purposes \( x \) days from the beginning of the growing season, \( 0 \leq x \leq 90 \).

Use this information to answer the next four questions.

7. Find the average value of \( W(x) \) from \( x = 0 \) to \( x = 90 \).
   a. 11502  b. 10783  c. 127.8  d. 195.3

8. Find the average rate of change of \( W(x) \) from \( x = 0 \) to \( x = 90 \)?
   a. 4.1  b. 369  c. 379.68  d. 7.59

9. Interpret \[ \frac{\int_{0}^{30} W(x) \, dx}{30} = 19.8 \] .
   “During the first 30 days of the growing season, ___________________________”
   a. the average amount of water available for irrigation increased by 19.8 hundred gallons.
   b. there was enough water available to irrigate the crops for an average of 19.8 days.
   c. there was an average of 19.8 hundred gallons of water available for irrigation.
   d. the amount of water available for irrigation increased by an average of 19.8 hundred gallons per day.

10. Interpret \[ \frac{\int_{0}^{30} W'(x) \, dx}{30} = 1.1 \] .
    “During the first 30 days of the growing season, ___________________________”
    a. the amount of water available for irrigation increased by an average of 1.1 hundred gallons per day.
    b. there was an average of 1.1 hundred gallons of water available for irrigation.
    c. there was enough water available to irrigate the crops for an average of 1.1 days.
    d. the average amount of water available for irrigation increased by 1.1 hundred gallons.
Weights of men are approximately normally distributed with a mean of 190 pounds and a standard deviation of 40 pounds. Federal requirements require FBI Police Officers to weigh between 117 and 238 pounds.

11. What proportion of men fall in this interval?
   a. 0.9145
   b. 0.8316
   c. 0.7912
   d. 0.8509

The life span of a compact fluorescent brand of light bulb has the probability density function

\[ f(t) = \begin{cases} 
\frac{24}{t^3} & \text{when } 3 \leq t \leq 6 \\
0 & \text{elsewhere}
\end{cases} \]

where \( t \) years is the length of the life span. [Check: \( f(4) = 0.375 \)]

Use \( f(t) \) to answer the next two questions.

12. What is the probability that a light bulb will have a life span less than 4.5 years?
   a. 0.2593
   b. 0.7407
   c. 0.5697
   d. 0.5000

13. The mean life span of this brand of light bulb is 4 years. Determine the standard deviation of the life span of this brand of light bulb.
   a. 0.1631 years
   b. 0.4039 years
   c. 0.6355 years
   d. 0.7972 years
Delta Airlines quotes a flight time of 125 minutes for its flights from Cincinnati to Tampa. Assume actual flight times are uniformly distributed between 121 minutes and 139 minutes. Use this information to answer the next two questions.

14. What is the mean flight time for a flight from Cincinnati to Tampa?
   a. 125 minutes
   b. 129.7 minutes
   c. 131.2 minutes
   d. 130 minutes

15. What is the probability that the flight will be more than 9 minutes late?
   a. 0.2222
   b. 0.2778
   c. 0.5000
   d. 0.3849

Between 1956 and 2000, the rate of change of the winning times for the 100-meter butterfly at a world-class swimming competition can be described by \( w(t) = 0.0106t - 1.146 \) seconds per year, where \( t \) is the number of years since 1900.

16. Find the average rate of change of the winning times for the competition from 1956 through 2000.
   a. –14.045 seconds per year
   b. –0.319 seconds per year
   c. 0.011 seconds per year
   d. 0.008 seconds per year

Check your Scantron now to make sure it will successfully run. If it does, you will earn one point. (1 pt)

When you are not working on the multiple choice portion of the test, turn your Scantron over so that it cannot be read by others in the room.
RE-READ the directions at the beginning of the test. Then read each question carefully. Provide only one clearly indicated answer to each question. If your answer is illegible, it will be graded as incorrect. Show all work. This portion is 51%.

For each question, set up the **specific mathematical notation** that is being evaluated to obtain your answer. No credit will be awarded for simply copying generic formulas from the formula sheet.

1. Let the random variable $x$ represent the time (in minutes) it takes a mouse to pass through a maze in a psychology experiment. Suppose the probability density function for $x$ is given by

$$f(x) = \begin{cases} 
5x^{-2} & \text{when } x \geq 5 \\
0 & \text{when } x < 5
\end{cases}.$$

Use the **algebraic method for evaluating an improper integral** to find $P(x \geq 9)$. That is, find

$$\int_{9}^{\infty} f(x) \, dx.$$ Show all of your steps and use proper notation throughout your work. (8 pts)

$$\int_{9}^{\infty} f(x) \, dx = \lim_{N \to \infty} \int_{9}^{N} 5x^{-2} \, dx$$

$$= \lim_{N \to \infty} \left[ -5x^{-1} \right]_{9}^{N}$$

$$= \lim_{N \to \infty} \left( -5(N)^{-1} - (-5(9)^{-1}) \right)$$

$$= \lim_{N \to \infty} \frac{-5}{N} + \lim_{N \to \infty} \frac{5}{9}$$

$$= (0) + \frac{5}{9}$$

$$= \frac{5}{9}$$

$$\approx 0.556$$

2 pts correctly writing as the limit of the integral with N or the limit of the antiderivative evaluated from a to N

2 pts correct anti-derivative (All or nothing)

1 pt substitution & subtraction (follow work)

(upper – lower)

1 pt limit of their algebraic (N) term (did not have to show if it was really 0)

1 pt limit of their constant term (must be correct based on their anti-derivative)

1 pt final answer (fraction or decimal is ok; only award this point for the correct answer; must follow from the work that is shown)

Students may show fewer steps as long as the work is clear and correct.

**Deductions**

-0.5pt each for notational & algebraic errors (up to a max of 1pt)

Ex: dropping limit too soon, missing vertical bar, limit on the wrong side of equals sign, infinity plugged in as if it were a number, missing dx, keeping the integral or dx after the antiderivative is found, etc.

No credit was awarded for answers obtained:

- using the numerical method
- using the property that $P(x > a) = 1 - P(0 < x < a)$
2. The demand for single bedroom apartments in a small college town is given by

\[ D(p) = -0.001p^2 - 0.2p + 466.1 \] apartments when the monthly rent is $p$ per apartment.

[Check: \( D(375) = 250.475 \)]

a. On the graph shown to the right, shade the region that represents the amount consumers are willing and able to spend when 200 apartments are rented.

**Find and label all** prices and quantities that are used to find this amount. (5 pts)

- \( p_0 = 425.4521862 \)
- \( p_{\text{max}} = 590 \)

2 pts for each value (rounded to two or unrounded)
1 pt for correct shading
Deduct up to 1 pt for truncated values

b. Find the Consumers’ Willingness and Ability to Spend when 200 apartments are rented. Round to two decimal places and include units. (4 pts)

\[ \text{CWAS} = \text{CE} + \text{CS} \]

\[
\begin{align*}
\text{CWAS} &= p_0 q_0 + \int_{p_0}^{p_{\text{max}}} D(p) \, dp \\
&= \left( \frac{\$425.45}{\text{apartment}} \right)(200 \text{ apartments}) + \int_{425.45}^{590} D(p) \, dp \\
&= \$85090 + \$17197.77 \\
&= \$102,287.77
\end{align*}
\]

If the unrounded \( p_0 \) is used, \( \text{CE} = 85090.43724, \text{CS} = 17197.33035, \text{CWAS} = 102,287.77 \)

1 pt CE
1.5 pts CS
1 pt CWAS
½ pt units
Incorrect values from part a were followed when work was shown.

B. Given \( D'(p) = -0.002p - 0.2 \), find the price at the point of unit elasticity. Round to two decimal places and include units. 2.5 pts number, ½ pt units; deduct ½ pt for notational errors such as equating \( \eta \) and \( p_0 \). 1 pt partial credit was awarded if the correct set up was shown. (3 pts)

\[
\begin{align*}
|\eta| &= \left| \frac{pD'(p)}{D(p)} \right| = \left| \frac{p(-0.002p-0.2)}{-0.001p^2-0.2p+466.1} \right| = 1 \text{ when } p_0 = \$333.10 \text{ per apartment} \\
\text{OR } \eta &= \frac{pD'(p)}{D(p)} = \frac{p(-0.002p-0.2)}{-0.001p^2-0.2p+466.1} = -1 \text{ when } p_0 = \$333.10 \text{ per apartment}
\end{align*}
\]

c. Determine if the demand for single bedroom apartments in the small college town is elastic or inelastic at a price of $350 per apartment. Show the specific value that justifies your answer. (3 pts)

\[
|\eta(350)| = 1.151 \quad 2 \text{ pts; deduct ½ pt for negative}
\]

Conclusion: The demand for single bedroom apartments is **elastic** when the monthly rent is $350 per apartment. 1 pt answer that follows work
3. Suppose that the demand for an adult Darth Vader costume during the Halloween season at a nationwide store can be modeled by 
\[ D(p) = 400(0.98^p) \] thousand costumes where \( p \) dollars per costume is the price of a costume. 

The supply of these Darth Vader costumes can be modeled by 
\[ S(p) = \begin{cases} 
0 & \text{for } p < 28 \\
0.93p + 30.855 & \text{for } p \geq 28 
\end{cases} \] thousand costumes where \( p \) dollars per costume is the price of a costume. 

\[ \text{[Check: } D(2) = 384.16 \] 

\[ \text{[Check: } S(50) = 77.355 \]

\[ \text{thousand costumes} \]

a. What is the minimum price that the producers are willing to accept for a costume? Include units. 
\[ \text{$28/costume} \] (3 pts) 
2.5 pts number, ½ pt units

b. At what price is the market at equilibrium? Round your answer to two decimal places and include units. 
\[ \text{$70.45/costume} \] (3 pts) 
2.5 pts number, ½ pt units; deduct ½ pt for rounding errors

c. Write the specific mathematical notation that could be used to find the total social gain at market equilibrium. Shade the region that represents the total social gain on the graph above. **DO NOT** find this amount. 
\[ \int_{28}^{70.45} S(p)dp + \int_{70.45}^{\infty} D(p)dp \] (6 pts) 
1 pt shading (values do not have to be indicated) 
2 pts for the SUM of the specific surpluses 
1 pt for \( p_s \) (follow work from part a) 
1 pt for \( p^* \) (1/2 each) (follow work from part b; rounded or unrounded value is ok) 
1 pt for infinity 
Deduct ½ pt for one or two missing dps 
1 pt partial credit was awarded if the correct, generic formula was shown
4. Four years ago, a company had profits of 390 thousand dollars and the company started depositing a half of their profits into an investment account earning interest at an annual rate of 1.99% compounded continuously. The profits increased by 2.1% each year for the next four years. Assume a continuous income stream.

a. Write $R(t)$, the income stream function. Include units. 

$$R(t) = 0.5 \left[ 390 \left(1.021^t \right) \right] \text{ OR } R(t) = 195 \left(1.021^t \right) \text{ thousand dollars per year}$$

1/2 pt for 0.5, 1 pt for any exponential function, 1.5 pts for $a$, ½ pt units 
Deduct 2 pts for including $e^{\text{anything}}$ in $R(t)$; deduct ½ pt for notational errors.

b. Assuming no additional deposits or withdrawals, how much money is in the account today? Round your answer correct to three decimal places and include units.

$$\int_0^4 R(t)e^{0.0199(4-t)} \, dt = 846.119 \text{ thousand dollars or } $846,118.58$$

1.5 pts for the correct setup of the future value integral (i.e. not the present value, and nothing plugged in for $t$); ½ pt for correctly identifying $T$, ½ pt for correctly identifying $r$, 1 pt for the correct answer (do not follow incorrect $R(t)$ functions), ½ pt units; accept 846.119 or 846.118. Deduct ½ pt for missing $dt$ notation. Note: If the present value is found, 1.5 pts is the max score.

5. Is $f(x) = \begin{cases} 2\sqrt{4-x} & \text{ when } 0 \leq x \leq 2 \\ 0 & \text{ elsewhere} \end{cases}$ a valid probability density function? If $f$ is a valid pdf, show how the conditions are satisfied. If $f$ is not a valid pdf, identify the condition(s) that is (are) not satisfied. Show specific notation/values in your justification.

No, $f(x)$ is not a valid pdf because $\int_{-\infty}^{\infty} f(x) \, dx = \int_{0}^{2} f(x) \, dx = 6.895 \neq 1$.

1 pt no, 3 pts for the SPECIFIC reason (the specific integral and its value should be shown).

6. A medical research team has determined that for 28-year old females, the length of pregnancy from conception to birth varies according to a normal distribution with a mean of 262 days and a standard deviation of 18 days.

**According to the Empirical Rule**, what percentage of 28-year old females will have a pregnancy that lasts more than 298 days? Include a well-labeled sketch in your work. (4 pts)

$$P(x < 234) = \frac{1 - 0.95}{2} = 0.025 \Rightarrow 2.5\%$$

1 pt sketch (262 and 298 labeled) 
3 pts answer