You are permitted to use a calculator on all portions of this test. You are not allowed to use any textbook, notes, cell phone, or laptop on any portion of this test. All devices must be turned off while you are in the testing room.

During this test, any communication with any person (other than the instructor or test proctor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.

No part of this test may be removed from the testing room.

Read each question very carefully. In order to receive full credit for the free response portion of the test, you must:
1. Show legible and logical (relevant) justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.

You have 90 minutes to complete the entire test.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student’s Signature: ________________________________

Do not write below this line.

<table>
<thead>
<tr>
<th>Free Response Problem</th>
<th>Points Possible</th>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td></td>
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<td>4</td>
<td>10</td>
<td></td>
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<td>Free Response Total</td>
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<td></td>
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<tr>
<td>Multiple Choice</td>
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<td></td>
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<tr>
<td>Correct Scantron</td>
<td>1</td>
<td></td>
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<tr>
<td>Test Total</td>
<td>100</td>
<td></td>
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</tbody>
</table>
Part I: Multiple Choice. There are 20 multiple choice questions. Solve each question using the available space for scratch work. Decide which is the best of the choices given and fill in the corresponding oval on the provided scantron using a #2 pencil. For your own record, also circle your choice on your test since the scantron will not be returned to you. Only the responses recorded on your scantron sheet will be graded. Each multiple choice question is worth 3 points.

1. Find \( P(Z > 0.05) \).
   (A) – 1.645
   (B) 0.4801
   (C) 0.5199
   (D) 1.645

2. The sample proportion, \( \hat{p} \), is an estimator of what?
   (A) \( \mu \)
   (B) \( \bar{x} \)
   (C) \( \sigma^2 \)
   (D) \( p \)

3. A random sample of 50 college students is surveyed and asked which of the 15 candidates that will appear on the ballot in the upcoming South Carolina Republican presidential primary they would most like to see win the Republican nomination for president. Is this an example of a binomial experiment? If not, state why not.
   (A) Yes, this is an example of a binomial experiment.
   (B) No, because the number of trials is not fixed.
   (C) No, because the trials are not independent.
   (D) No, because there are more than two possible outcomes in each trial.

4. Find the value of \( z_{10} \).
   (A) – 1.28
   (B) 0.5398
   (C) 1.28
   (D) 2.33
5. Below is a normal probability plot for a random sample of $n = 30$ observations.

Based on the normal probability plot, is it reasonable to believe that the sample data are from a population that is normally distributed?

(A) Yes, because the normal probability plot is roughly linear and all points are inside the bounds.
(B) No, because the normal probability plot is roughly linear and all points are inside the bounds.
(C) Yes, because there is a curved pattern to the normal probability plot and some points are outside the bounds.
(D) No, because there is a curved pattern to the normal probability plot and some points are outside the bounds.

6. Find the area under the standard normal curve that lies between $Z = -0.09$ and $Z = 2.52$.

(A) 0.5300
(B) 0.9941
(C) 0.9955
(D) 1.4582
Use the following information to answer questions 7 – 8.

Eight percent of men cannot distinguish between the colors red and green. This is the type of color blindness that causes problems with traffic signals. Suppose fifteen men are randomly selected for a study of traffic signal perceptions and let \( X \) = the number of them that cannot distinguish between red and green. In the JMP output below, the first column gives the possible values of \( X \), the second column gives the binomial probabilities \( P(X = x) \), and the third column gives the cumulative binomial probabilities \( P(X \leq x) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>( p(x) )</th>
<th>Cumulative Probabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.2863</td>
<td>0.2863</td>
</tr>
<tr>
<td>1</td>
<td>0.3734</td>
<td>0.6597</td>
</tr>
<tr>
<td>2</td>
<td>0.2273</td>
<td>0.887</td>
</tr>
<tr>
<td>3</td>
<td>0.0857</td>
<td>0.9727</td>
</tr>
<tr>
<td>4</td>
<td>0.0223</td>
<td>0.995</td>
</tr>
<tr>
<td>5</td>
<td>0.0043</td>
<td>0.9993</td>
</tr>
<tr>
<td>6</td>
<td>0.0006</td>
<td>0.9999</td>
</tr>
<tr>
<td>7</td>
<td>0.0001</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>12</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>14</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

7. What is the probability that among fifteen randomly selected men, more than two of them cannot distinguish between red and green?

(A) 0.1130

(B) 0.2273

(C) 0.3403

(D) 0.8870

8. Find the mean and standard deviation of \( X \).

(A) \( \mu = 1.2, \sigma \approx 1.051 \)

(B) \( \mu = 1.2, \sigma = 1.104 \)

(C) \( \mu = 12, \sigma \approx 1.549 \)

(D) \( \mu = 12, \sigma = 2.4 \)
9. The length of time it takes for a family of 4 to play 18 holes of miniature golf is approximately normally distributed with mean 42 minutes and standard deviation 10 minutes. What percentage of families take longer than an hour to play 18 holes of miniature golf?

(A) 4%
(B) 18%
(C) 22%
(D) 96%

10. Which of the following is a discrete random variable?

(A) The weight (in ounces) of a McDonald's hamburger
(B) The amount of fat (in grams) in a McDonald's hamburger
(C) The time it takes to cook a McDonald's hamburger
(D) The number of McDonald's hamburgers sold in a 24 hour period

11. According to motortrend.com, white is the most popular car color with 23% of all cars sold in 2014 being white. If 10 people that bought a car in 2014 are randomly selected, what is the probability that exactly two of them purchased a white car?

(A) 0
(B) (.23)^2
(C) \(10 \choose 2\)(.23)^2(.77)^8
(D) \(10 \choose 2\)(.23)^8(.77)^2

12. Is the following distribution a legitimate discrete probability distribution? If not, state why not.

<table>
<thead>
<tr>
<th>X</th>
<th>P(X)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>0.24</td>
</tr>
<tr>
<td>0</td>
<td>0.31</td>
</tr>
<tr>
<td>3</td>
<td>0.19</td>
</tr>
<tr>
<td>6</td>
<td>0.16</td>
</tr>
<tr>
<td>9</td>
<td>0.10</td>
</tr>
</tbody>
</table>

(A) Yes, this distribution is a legitimate discrete probability distribution.
(B) No, because at least one of the probabilities is less than 0 or greater than 1.
(C) No, because one of the possible outcomes is negative.
(D) No, because the sum of the probabilities is not equal to 1.
13. The probability distribution of the random variable $X$ is given below.

<table>
<thead>
<tr>
<th>$X$</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>P($X$)</td>
<td>0.10</td>
<td>0.25</td>
<td>0.45</td>
<td>0.15</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Which of the following is NOT true?

(A) $P(X = 2) = 0.25$

(B) $P(X = 0) = 0$

(C) $P(X < 4) = 0.95$

(D) $P(X \geq 2) = 0.90$

14. Find the two Z values that separate the middle 98% of the standard normal distribution from the area in the tails of the standard normal distribution.

(A) – 0.8365 and 0.8365

(B) – 1.75 and 1.75

(C) – 2.05 and 2.05

(D) – 2.33 and 2.33

15. A researcher is interested in knowing the proportion of all Clemson University students that are biology majors. She takes a simple random sample of 50 Clemson University students, and finds that 20 are biology majors. Which of the following statements is true?

(A) $\hat{p} = .4$

(B) $\hat{p} = 20$

(C) $p = .4$

(D) None of the above statements is true.
16. The number of hours college students sleep the night before finals week begins has a distribution that is quite skewed. Let \( \bar{x} \) represent the mean number of hours slept the night before finals week begins for a random sample of 45 college students. What can be said about the sampling distribution of \( \bar{x} \)?

(A) The sampling distribution of \( \bar{x} \) may not be normally distributed because the number of hours college students sleep the night before finals week begins is not normally distributed.

(B) The sampling distribution of \( \bar{x} \) is approximately normally distributed because hours of sleep is a continuous random variable.

(C) The sampling distribution of \( \bar{x} \) is approximately normally distributed because the sample size is large enough.

(D) The sampling distribution of \( \bar{x} \) may not be normally distributed because the sample size is not large enough.

17. In a population of college students, the number of lab classes taken this semester is a random variable \( X \) with the following probability distribution.

<table>
<thead>
<tr>
<th>( X )</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P(X) )</td>
<td>0.1</td>
<td>0.6</td>
<td>0.3</td>
</tr>
</tbody>
</table>

Find the standard deviation of \( X \).

(A) 1.20

(B) 0.60

(C) 0.36

(D) Cannot be determined from the given information.
Use the following information to answer questions 18 – 19.
In 2014, composite ACT scores were approximately normally distributed with mean 21.0 and standard deviation 5.4.

18. A very prestigious private college considers only students who scored in the top 2% of those taking the ACT. What ACT score is necessary to be accepted to this college?
   (A) 21
   (B) 24
   (C) 26
   (D) 32

19. Which of the following statements about 2014 ACT scores is false?
   (A) A score of 15 is about 1.11 standard deviations below the mean
   (B) The median score is 21
   (C) The 90th percentile of the 2014 ACT distribution is 30
   (D) Approximately 95% of students taking the test score higher than 12

20. Which of the following statements about the parameters of the sampling distribution of the sample mean is true for samples of size $n = 40$?
   (A) The mean of the sampling distribution is larger than the population mean.
   (B) The mean of the sampling distribution is smaller than the population mean.
   (C) The standard error of the sampling distribution is larger than the population standard deviation.
   (D) The standard error of the sampling distribution is smaller than the population standard deviation.
Part II: Free Response. Show all your work. Indicate clearly the methods you use, because you will be graded on the correctness of your methods as well as on the accuracy and completeness of your results and explanations. Answers with no justification will receive no credit.

1. One of the games at a carnival is Spin the Pointer, which uses a spinner like the one pictured below.

A carnival ticket that costs $1 is required to play the game. For each $1 ticket, a player spins the pointer once and receives the amount of money indicated in the sector where the pointer lands on the wheel. The spinner has an equal probability of landing in each of the 8 sectors.

(a) Let $X$ represent the profit for the player from one play of the game. Complete the table below for the probability distribution of $X$. (3 pts)

<table>
<thead>
<tr>
<th>$X$</th>
<th>$-1$</th>
<th>$0$</th>
<th>$4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(X)$</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(b) Find the expected value of the profit for the player from one play of the game. (3 pts)

(c) Interpret the expected value calculated in part (b). (3 pts)
2. Delivery time for Frank's Pizza is approximately normally distributed with mean 23 minutes and standard deviation 4.6 minutes.

   (a) If you order a pizza from Frank's, what is the probability that your pizza will arrive between 15 minutes and 20 minutes after you place your order? Show all your work, label your answer with appropriate probability notation, and give the answer to four decimal places. (5 pts)

   (b) Frank wants to guarantee delivery in a certain amount of time or less or the pizza is free. If Frank does not want to give away pizzas more than 2.5% of the time, what time should he advertise as the maximum delivery time before the pizza is free? Show all your work and give the answer to the nearest minute. (5 pts)
3. An intelligence quotient, or IQ, is a measurement of intelligence derived from a standardized test, such as the Stanford-Binet IQ test. Scores on this test are normally distributed with a mean of 100 and a standard deviation of 15. An individual with an IQ score of 120 or above is classified as having "superior" intelligence.

(a) What is the probability that a randomly selected individual has an IQ score of 120 or above and is thus classified as having "superior" intelligence? Show all your work, label your answer with appropriate probability notation, and give the answer to four decimal places. (4 pts)

(b) Let $\bar{x}$ represent the mean IQ score for a random sample of 5 individuals. Why is the sampling distribution of $\bar{x}$ approximately normal? (2 pts)

(c) What is the probability that a random sample of 5 individuals has a mean IQ score of 120 or above? Show all your work, label your answer with appropriate probability notation, and give the answer to four decimal places. (4 pts)
4. According to climatological experts, it is known that it will rain on 30% of summer days in a certain city. Let \( \hat{p} \) represent the proportion of rainy days in a simple random sample of 100 summer days in this city.

(a) The sampling distribution of \( \hat{p} \) is approximately normal. Verify that this is the case using the rule of thumb we used in class. (2 pts)

(b) Give the mean and standard deviation of the sampling distribution of \( \hat{p} \). Round your answer for the standard deviation to five decimal places. (2 pts)

(c) What is the probability that there are 18 or fewer rainy days in a simple random sample of 100 summer days in this city? Show all your work, label your answer with appropriate probability notation, and give the answer to four decimal places. (4 pts)

(d) If you actually obtain a simple random sample of 100 summer days in this city in which 18 or fewer of them are rainy, would this cause you to suspect that the climatological experts are overestimating the percentage of days with rain? Explain your answer. (2 pts)