1. A dairy uses process machinery to package milk into individual quart-sized coated paper containers. The machine fills a continuous coated paper tube with milk, seals and cuts the tube between the quarts, folds the containers, and pallets the quarts of milk for shipment to stores.

On a particular day, the process machinery began to package the milk at a rate of 400 quarts per minute. After 4 minutes machine speed was steadily increased, so that after 2 additional minutes the machine was processing 960 quarts per minute. The machine continued at this rate until the milk was packaged, which took a total of 23 minutes, as shown in the graph below.

![Graph showing milk packaging rate over time]

How much milk was packaged in the 23 minutes?

A. 19,280 quarts  
B. 17,920 quarts  
C. 18,480 quarts  
D. 21,760 quarts  
E. 22,080 quarts

2. \( r(x) = 765x^2 - 232x + 459 \) gallons per hour is a rate of change function (check: \( r(2) = 3055 \)). If \( R(x) \) is an antiderivative of \( r(x) \), and it is known that \( R(4) = 30,068 \), compute \( R(1) \).

A. 14499 gallons  
B. 14388 gallons  
C. 14455 gallons  
D. 14377 gallons  
E. 14366 gallons
3. \( r(x) = 765x^2 - 232x + 459 \) gallons per hour is a rate of change function (check: \( r(2) = 3055 \)).

Find the average value of \( r(x) \) on the interval \(-2 \leq x \leq 2\).

A. 1479 gallons per hour  
B. 5916 gallons per hour  
C. 1386 gallons per hour  
D. 4964 gallons per hour  
E. 1266 gallons per hour

4. Given \( P(x) = 45x^3 + 21x^2 + 15x \) pounds, where \( x \) equals the number of hours since 4:00 pm (check: \( P(3) = 1449 \)). Find the average rate of change of \( P(x) \) on the interval \( 2 \leq x \leq 6 \).

A. 4024 pounds per hour  
B. 2523 pounds per hour  
C. 1386 pounds per hour  
D. 4964 pounds per hour  
E. 1266 pounds per hour

5. \( f(x) = -0.2x^3 + 4x^2 - 6x - 20 \) is graphed to the right. Check: \( f(2) = -17.6 \)

Use two midpoint rectangles to estimate the signed area between \( f(x) \) and the \( x \)-axis on the interval \( 1 \leq x \leq 9 \).

A. 15.6  
B. 11.2  
C. 232  
D. 516.8  
E. 242.7
6. \( f(x) \) is a continuous function. \( \frac{d}{dp} \left( \int_1^4 f(x) \, dx \right) =
\)

A. \( F(p) \)
B. \( f(x) \)
C. \( F(x) \)
D. \( f(p) \)
E. All of the above may be considered correct.

7. The graph shows a **rate of change** function. Point \( a \) corresponds to what type of feature on a related accumulation function?

A. It is a relative minimum
B. It is a relative maximum
C. It is an inflection point
D. It does not correspond to anything

8. The graph shows an **accumulation** function. Point \( a \) corresponds to what type of feature on a related rate of change function?

A. It is a relative minimum
B. It is a relative maximum
C. It is an inflection point
D. It does not correspond to anything

9. Find the value of the following expression \( \int_1^4 (3x^3 - 4x^2 + 5x - 2) \, dx \)

A. 144.25
B. 38.75
C. 138.75
D. 32.25
E. 56.50
10. Given the following information:

\[ \frac{d}{dx} J(x) = j(x) ; \quad j(2) = 8; \quad j(10) = 12; \quad \int_{2}^{10} j(x) \, dx = 22 \quad \text{and} \quad J(2) = 10. \] Find \( J(10) \), if possible.

A. \( J(10) = 30 \)
B. \( J(10) = 34 \)
C. \( J(10) = -4 \)
D. \( J(10) = 32 \)
E. It is not possible to find \( J(10) \) with the information given.

11. The height of bamboo plant is \( P(h) = 0.042h^3 - 0.13h^2 + 7h + C \) inches, where \( h \) is the number of hours after 12:00 noon. At 2:00 pm this plant was 16 inches tall. How tall was this plant at 9:00 pm?

A. 49.184 inches
B. 85.272 inches
C. 101.184 inches
D. 32.184 inches
E. 102.184 inches

12. \( d(t) \) is a rate of change function. \( D(x) = \int_{3}^{x} d(t) \, dt \). Which of the following statements is true?

A. \( D(3) = 0 \)
B. \( D(0) = 3 \)
C. \( D(x) \) is always increasing.
D. \( D(x) \) is always decreasing.
E. None of these statements are always true.

13. The function depicted is \( y = 2x - 0.8 \); find the sum of the signed areas trapped between \( y \) and the \( x \)-axis on the interval \(-2 \leq x \leq 2\).

A. -8.5
B. 4
C. 8.32
D. -5.76
E. -3.2
Mrs. Smith owns a hot dog stand.

Her accountant has supplied to her a function, \( h(x) \), which represents the rate of change of sales, measured in hotdogs per hour.

Mrs. Smith does not understand Calculus. Use the following to build a sentence of practical interpretation to explain to Mrs. Smith the meaning of the shaded area.

**QUESTIONS 14 – 17 INVOLVE WRITING A SINGLE SENTENCE. EACH QUESTION IS WORTH 1 POINT.**

14. Begin the best sentence of practical interpretation for the area trapped between \( h(x) \) and the \( x \)-axis on the interval of \( 4 \leq x \leq 7 \). The sentence will continue in subsequent questions.

   A. Between 2:00 pm and 5:00 pm.....
   B. The area under the curve between 4 and 7.....
   C. The integral of \( h(x) \) between \( x = 4 \) and \( x = 7 .....)
   D. From \( x = 4 \) to \( x = 7 .....)

15. Continue the best sentence of practical interpretation for the area trapped between \( h(x) \) and the \( x \)-axis on the interval of \( 4 \leq x \leq 7 \).

   A. …the decrease in the number of hot dogs sold…
   B. …the number of hot dogs sold…
   C. …the change in the number of hot dogs sold…..
   D. …the increase in hot dogs sold…..

16. Continue the best sentence of practical interpretation for the area trapped between \( h(x) \) and the \( x \)-axis on the interval of \( 4 \leq x \leq 7 \).

   A. …was increasing …
   B. …was decreasing…
   C. …increased…
   D. …decreased…

17. Finish the best sentence of practical interpretation for the area trapped between \( h(x) \) and the \( x \)-axis on the interval of \( 4 \leq x \leq 7 \).

   A. …to 210 hot dogs.
   B. …to 210 hot dogs per hour.
   C. …by 210 hot dogs.
   D. …by 210 hot dogs per hour.

This is the end of the multiple choice section of the test.
Free Response: The free response portion of the exam is worth 56 points. This portion will be hand graded. Answer all questions clearly as illegible work will be considered to be incorrect. Correct answers without the appropriate work shown will receive little or no credit. Where appropriate, for full credit, show mathematical notation, necessary work and units.

1. The graph above is of a rate of change function $f(x)$. $F(t) = \int_{k}^{t} f(x)dx$.

All questions below regard the accumulation function $F(t)$ on the interval $-2 < t < 9$ (only).

a. Circle the correct answers regarding the critical points:

$F(t)$ has a relative maximum at locations $a, b, d, e, f, g, h, i$

$F(t)$ has a relative minimum at locations $a, b, c, d, e, f, g, h, i$

$F(t)$ has an inflection point at locations $a, b, c, d, e, f, g, h, i$

1 point each: deduct 1 point for each incorrect response.

b. Fill in the correct answers regarding concavity on the interval $a \leq t \leq i$

(for example: if your answer is “between $b$ and $c$” write “$b < t < c$”).

You may not need all the spaces provided.

$F(t)$ is concave up in the following region(s): $b < t < e$ and $h < b < i$

$F(t)$ is concave down in the following region(s): $a < t < b$ and $e < b < h$

$\frac{1}{2}$ point each, all or nothing.
2. **Algebraically** compute the following integral. 
   Give the **exact** answer. 
   Do not represent values such as \(e^4\) or \(\ln 4\) with a decimal approximation. 
   **Combine like terms.** 
   Show each step necessary to find the answer (as points are awarded for each step in the process). 
   Show irrational values (such as \(2.33\)) as fractions (\(\frac{7}{3}\)).

\[
\int_2^6 (4x^2 - \frac{5}{x}) \, dx = \left[ \frac{4}{3} x^3 - 5 \ln x \right]_2^6 = \left[ \frac{4}{3} (6)^3 - 5 \ln (6) \right] - \left[ \frac{4}{3} (2)^3 - 5 \ln (2) \right]
\]

2 points for antiderivative with bar and limits of integration
1 ½ points for each bracket with limits plugged in. If reversed, only ½ point total

\[
= \left[ 288 - 5 \ln 6 \right] - \left[ \frac{32}{3} - 5 \ln 2 \right]
\]

\[
= \frac{832}{3} - 5(\ln 6 - \ln 2) \quad \text{or} \quad 277\frac{1}{3} - 5(\ln 6 - \ln 2)
\]

no points for intermediate simplification

½ point for each term in the final answer.
Deduct 1 point if student does not use equal signs (or uses arrows between steps).
No credit for a decimal equivalent of the answer.

- ½ for leftover C
- ½ for not combining like terms in the final answer
- ½ for incorrect sign in the final answer
- 1 for two or more missing equal signs

Full credit for alternative first step \(F(b) - F(a)\) where \(F(x)\) is defined correctly.
3. The rate of change of the weight of a laboratory mouse can be modeled as
\[ w(t) = \frac{13.785}{t} \] grams per week, where \( t \) equals the number of week after the beginning of an experiment, \( 1 \leq t \leq 15 \).
Find the change in the weight of the mouse between the third and ninth weeks. Show your work with mathematical notation.

\[ \int_{3}^{9} w(t) \, dt = 15.1443 \] 1 point for correct notation. No credit for ‘Y_i’

Answer: The mouse \( \underline{gained} \), \( 15.144 \) grams.

1 point for ‘gained’, 1 point for correct quantity of weight gain.
4. You are given the following:
   a. $G(x)$ is an antiderivative of $g(x)$.
   b. $G(a) = -272$
   c. $G(b) = 326$

   
   Either: find $\int_a^b g(x) \, dx = \boxed{598}$
   3 points for correct answer. No work necessary

   Or: Explain (in less than 25 legible words) why it is not possible to do so with the given information.

5. You are given the following:
   a. During a storm, snow was falling in Boston at a rate of $b(x) = 8(0.76^x)$ inches per hour,
      where $x$ is the number of hours since 6:00 am, $1 \leq x \leq 6$.
   b. At 8:00 am the snow was falling at the rate of 4.62 inches per hour.
   c. The function $B(x)$ is an antiderivative of $b(x)$.

   Either: find the amount of snow on the ground at 11:00 am.

   Or: Explain (in less than 25 legible words) why it is not possible to do so with the given information.

   We do not know how much snow was on the ground at 11:00 am because we never were given any
   snow accumulation values. The constant C could not be established.

   Give 3 points all or nothing. If the answer is not adequate, give no credit.
6. Find the following. Simplify expressions when possible. *Place a box around your final answers.*

a. \[ \int (8x^3 + 21x^2 + 10x) \, dx = 2x^4 + 7x^3 + 5x^2 + C \] 1 point for each numerical term. -1 if no C

b. \[ \int \left[ \frac{d}{dt}(6t^3 - 9t^2 + 6t - 14) \right] \, dt = 6t^3 - 9t^2 + 6t + C \] 1 point for each numerical term. -1 if no C

-1 if “14” instead of “C”.

c. \[ \int \left[ \frac{1}{6x} + e^{3x} + 7x + 5\pi^3 \right] \, dx = \frac{1}{6} \ln |x| + \frac{e^{3x}}{3} + \frac{7x}{\ln 7} + 5\pi^3 x + C \] 1 point for each numerical term. -1 if no C

d. \[ \frac{d}{dt} \int_4^t (\frac{16}{x} + 32.7x + 15e^{4t}) \, dx = \frac{16}{t} + 32.7t + 15e^{4t} \] 1 point for each numerical term. -1 if “+ C”
7. Given two functions, \( f(x) = 2x - 1 \) and \( g(x) = x^2 - 4 \), pictured below.

Find the area trapped between the two curves on the interval \(-1 \leq x \leq 3\).

Show the appropriate mathematical notation to compute this area.

Give the EXACT answer (hint: if your answer is irrational, show your answer as a fraction).

\[
\int_{-1}^{3} [f(x) - g(x)] \, dx = 10 \frac{2}{3}
\]

3 points for mathematical notation.
- Deduct one point if the interval is not shown.
- Deduct one point if the functions are reversed.
- Deduct \( \frac{1}{2} \) point if no \( dx \) is included.

1 point for correct numerical answer. If shown as 10.666, deduct \( \frac{1}{2} \) point.
8. The rate of change of profit for an on-line business is given by:

\[ f(x) = -55x^3 + 795x^2 - 2958x + 2962 \text{ thousand dollars/year}, \]

where \( x \) is the number of years since 1999, \( 1 \leq x \leq 10 \).

In the year 2000, the rate of change of profit was 744 thousand dollars per year.
In the year 2004, the profit for the business was 2632 thousand dollars.


\[ \frac{\int_{1}^{10} f(x)\,dx}{9} = \frac{831.75}{9} \text{ thousand dollars/year} \]

3 points for the mathematical notation, 2 points for the quantity, 1 point for the units.
- ½ for rounding error.
- 1 if calculator notation was used instead of mathematical notation.
- 1 if using the incorrect interval.
- 1 for not writing the division by denominator regardless of correct solution.
- 2 for incorrect denominator.

b. Recover the function \( F(x) \), the profit for the business. That is, find the specific antiderivative \( F(x) \).

\[ F(x) = -13.75x^4 + 265x^3 - 1479x^2 + 2962x + 265.75 \text{ thousand dollars} \]

½ point for each numerical term, 2 points for the \( C \), ½ point for the units.
1 point partial credit for \( C \) was awarded if relevant work was shown.
- ½ if \( F(x) \) not restated with specific value of \( C \) plugged in.
- 1 if \( F(x) \) is never stated (i.e., if \( C \) is found without ever stating \( F(x) \) explicitly).

then, find the profit in 2006: \[ F(7) = 6410 \text{ thousand dollars} \] 1.5 points include units.

c. What was the change in profit between 2003 and 2007? Show mathematical notation, and include units.

\[ \int_{4}^{8} f(x)\,dx = 6776 \text{ thousand dollars or } F(8) - F(4) = 6776 \text{ thousand dollars} \]

1 ½ points for mathematical notation, 1 point for correct answer, ½ points for units.
- 1 if calculator notation is used instead of mathematical notation.
- ½ for incorrect interval.
- 1 for \( F(a) - F(b) \).
- ½ for using \( F \) in the integral notation if \( f \) was actually integrated.