You are **not** permitted to use a calculator on any part of this test. You are **not** allowed to use any textbook, notes, cell phone, laptop, PDA, or any technology on any part of this test. All devices must be turned off while you are in the testing room.

During this test, any communication with any person (other than the instructor or his designated proctor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.

No part of this test may be removed from the testing room.

Read each question very carefully. In order to receive full credit, you must:

1. Show legible and logical (relevant) justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.
4. Give exact numerical values whenever possible.

You have **90 minutes** to complete the entire test.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student’s Signature: __________________________

Do not write below this line.

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Read each question carefully. In order to receive full credit you must show legible and logical (relevant) justification which supports your final answer. Give answers as exact answers. You are NOT permitted to use a calculator on any portion of this test.

1. (4 pts.) If the tangent line to $y = f(x)$ at $(4, 3)$ passes through the point $(0, 2)$, find $f(4)$ and $f'(4)$.

2. (5 pts.) Sketch the graph of a function $f$ that satisfies all of the given conditions. Indicate any asymptotes with dotted line(s).

$$\lim_{x \to 0} f(x) = -2, \quad \lim_{x \to 2^-} f(x) = -1, \quad \lim_{x \to 2^+} f(x) = -2.5, \quad f(0) = 0, \quad f \text{ is left continuous at } 2, \quad \lim_{x \to \pm\infty} f(x) = 1$$

3. (6 pts.) Suppose a stone is tossed vertically upward. The height of the stone (in feet above the ground) $t$ seconds after it is tossed is given by $s(t) = -16t^2 + 32t + 48$. Find the velocity of the stone at the instant it hits the ground. Be sure to include units with your answer.
4. Consider the function \( f(x) = \frac{x^3 - 4x^2 - 11x + 30}{x^2 - x - 12} = \frac{(x-2)(x+3)(x-5)}{(x+3)(x-4)} \).

a. (3 pts.) Find \( \lim_{x \to 4^+} f(x) \)

b. (1 pt.) What does your answer to part a tell you about \( f(x) \)?

Choose one: \( f(x) \) has a vertical asymptote at \( x = 4 \).
\( f(x) \) has a removable discontinuity at \( x = 4 \).

C. (4 pts.) Prove (with limits) that \( f \) does not have any horizontal asymptotes.

\textit{Hint: you will need two limits.}

5. (5 pts.) For which \( x \)-value(s) does the tangent line of \( f(x) = e^{3x} + 5 \) lie \textbf{perpendicular} to the line \(-2x - 3y = 5\)?
6. (10 pts.) A paper cup has the shape of a cone with height 10 cm and radius 3 cm (at the top). If water is poured into the cup at a rate of $2 \text{ cm}^3$ per second, how fast is the water level rising when the water is 5 cm deep?

The volume of a right circular cone is $V = \frac{1}{3} \pi r^2 h$.

*Your answer must include a well labeled picture, a written description of the variables including units (use $t$ for time), a statement of what rate you are given, a general equation, the associated calculus and algebra, and a clearly stated solution with units. Work must be clear and logical to receive full points.*
7. (6 pts.) Use the limit definition of the derivative to show that the derivative of the function \( f(x) = 3x - 7x^2 \) is \( f'(x) = 3 - 14x \).

8. Find the derivative of the following functions. Use appropriate notation to denote the derivative. DO NOT SIMPLIFY the derivatives.

   a. (4 pts.) \( y = \cos^{-1}(\tan^{-1} t) \)

   b. (4 pts.) \( y = x^{5\pi} + e^x + \log_3 7x \)

   c. (5 pts.) \( y = \ln \left( g \left( \frac{2x + 3}{3x - 1} \right) \right) \) where \( g(x) \) is a differentiable function
9. (4 pts.) Evaluate the following limit. \( \lim_{{x \to 4}} \frac{x^3 - 64}{x^2 - 3x - 4} \)

10. (6 pts.) Find \( \frac{dy}{dx} \) for \( e^y = 1 + \sin(xy^2) \).

11. (4 pts.) Evaluate \( \lim_{{x \to 0}} \frac{7x}{\sin 3x} \). Step by step work MUST be shown. You will not be given any credit for using L’Hopital’s Rule.
12. A particle moves according to a law of motion \( s = f(t) = t^3 - 9t^2 + 15t + 10, \ t > 0 \), where \( s \) in meters and \( t \) in seconds.

a. (3 pts.) Find the velocity and acceleration functions at time \( t \). *Be sure to include units with your answer.*

b. (4 pts.) When is the particle moving in the negative direction?

c. (4 pts.) SET UP an expression that would help find the total distance traveled during the first 8 seconds. You do NOT need to take the time to evaluate the expression.

13. (6 pts.) If \( f(x) = (\ln x)^{\cos x} \), find \( f'(x) \).
14. Four months after it stops advertising, a manufacturing company finds that its sales have dropped from 100,000 units per month to 80,000 units per month. The sales follow an exponential pattern of decline.

*Be sure to include units with your answer to each part.*

a. **(4 pts.)** Find an expression for the amount of sales per month after \( t \) months. Give your final answer in terms of \( e \) and natural logarithms.

b. **(4 pts.)** When will the sales be 50,000 units per month? Give your answer using the natural logarithmic (ln) function.

15. **(4 pts.)** Evaluate the following limit. \( \lim_{x \to 5} \frac{\sqrt{3x+10} - 5}{x - 5} \)