[Topic 1] Applications of Integration

【6.1】Area between Curves

1. Consider the region below the graph of \( y = \arcsin \left( \frac{x}{2} \right) \) in the first quadrant bounded by \( x = 0 \) and \( x = 2 \).
   a. Set up, but **DO NOT EVALUATE**, the integral with respect to \( x \) that gives the area of the region.
   b. Set up, but **DO NOT EVALUATE**, the integral with respect to \( y \) that gives the area of the region.

2. Consider the region between the graphs of \( y = \cos x \) and \( y = \sin \left( 2x \right) \) from \( x = 0 \) to \( x = \frac{\pi}{2} \).
   Set up, but **DO NOT EVALUATE**, the integral(s) with respect to \( x \) that gives the area of the enclosed region.

3. Set up, but **DO NOT EVALUATE**, the integral(s) with respect to \( x \) that gives the area of the region in the first quadrant bounded by the axes, \( y = e^x \), \( x = e^y \) and the line \( x = 4 \).

【6.2】Volume by Slicing

1. The base of a certain solid is given by \( x^2 + y^2 = a^2 \), \( a > 0 \). Each cross section of the solid is a square with one side on the base of the solid. The cross sections are perpendicular to the \( x \)-axis. Find the volume of this solid.

2. A chamber is created by rotating the function \( y = a \sin x \), \( 0 \leq x \leq \pi \) about the \( x \)-axis. Find all values of \( a \) required for the chamber to hold exactly \( 2\pi^2 \) cubic units of sand.

3. Let \( R \) be the region bounded by \( y = e^{2x} \), \( y = 2 \) and \( x = 0 \) in the first quadrant.
   a. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the \( x \)-axis using the **disk/washer method**.
   b. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the \( y \)-axis using the **disk/washer method**.
c. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( x = \frac{1}{2} \ln 2 \) using the **disk/washer method**.

d. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( y = 2 \) using the **disk/washer method**.

4. Consider the region bounded by \( f(x) = 3x \) and \( g(x) = x^2 \).
   a. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( y = 10 \) using the **disk/washer method**.
   b. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( x = -1 \) using the **disk/washer method**.

【6.3】Volume by Shells

1. Follow #3 in 【6.2】
   a. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the \( x \)-axis using the **shell method**.
   b. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the \( y \)-axis using the **shell method**.

2. Let \( R \) be the region bounded by \( y = x^2 \), \( x = 2 \) and \( y = 0 \) in the first quadrant.
   a. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( x = -5 \) using the **shell method**.
   b. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( x = 5 \) using the **shell method**.
   c. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( y = -5 \) using the **shell method**.
   d. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( y = 5 \) using the **shell method**.
3. Follow #4 in 【6.2】
   a. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( y = 10 \) using the shell method.
   b. Set up, but do not evaluate or simplify the integral that gives the volume of the solid obtained by rotating the region \( R \) around the line \( x = -1 \) using the shell method.

【8.1】Arc Length
1. We can approximate the length of a curve with the sum of the lengths of line segments, expressed as \( L \approx \sum_{k=1}^{n} \sqrt{\left(\Delta x\right)^2 + \left(\Delta y\right)^2} \). What theorem helps us rewrite this sum as \( L \approx \sum_{k=1}^{n} \sqrt{1 + \left[f'(x_k)\right]^2} \Delta x \)?
2. Find the equation of a curve that passes through the point \((1,5)\) and has an arc length on the interval \([2,6]\) given by \( \int_2^6 \sqrt{1 + 16x^{-6}} \, dx \).
3. Find the length of the curve \( y = \frac{1}{3}(x^2 + 2)^{\frac{3}{2}} \) for \( 0 \leq x \leq 1 \).
4. Find the length of the curve \( x = \frac{2}{3}y^\frac{3}{2} - \frac{1}{2}y^\frac{1}{2} \) for \( 1 \leq y \leq 9 \).

【8.2】Surface Area
1. Use Calculus to find the surface area of a sphere with radius \( r \).
2. Consider the arc of the curve \( y = \sqrt{1 + e^x} \) where \( 0 \leq x \leq 1 \). Find the area of the surface obtained by rotating this arc about the \( x \)-axis.
3. Consider the arc of the curve \( x = \frac{1}{3}(y^2 + 2)^{\frac{3}{2}} \) where \( 1 \leq y \leq 2 \). Find the area of the surface obtained by rotating this arc about the \( x \)-axis.
4. Consider the arc of the curve \( x = \sqrt{16 - y^2} \) where \( 0 \leq y \leq 2 \). Find the area of the surface obtained by rotating this arc about the \( y \)-axis.
5. Consider the arc of the curve \( y = \frac{1}{4}x^2 - \frac{1}{2}\ln x \) where \( 1 \leq x \leq 2 \). Find the area of the surface obtained by rotating this arc about the \( y \)-axis.
MTHS 1080 Final Exam Review Answer

[Topic 1] Applications of Integration

【6.1】Area between Curves

(1) (a) \( \int_0^2 \arcsin \left( \frac{x}{2} \right) \, dx \) (b) \( \int_0^{\frac{\pi}{2}} \left( 2 - 2 \sin y \right) \, dy \) (2) \( \int_0^\pi \left( \cos x - \sin 2x \right) \, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left( \sin 2x - \cos x \right) \, dx \)

(3) \( \int_0^1 e^x \, dx + \int_1^4 \left( e^x - \ln x \right) \, dx \)

【6.2】Volume by Slicing

(1) \( \frac{16}{3} a^3 \) cubic units (2) \( \pm 2 \) (3) (a) \( \int_0^{\ln 2} \frac{\pi}{2} \left[ 2^2 - \left( e^{2x} \right)^2 \right] \, dx \) (b) \( \int_0^\pi \frac{\pi}{2} \left( \frac{1}{2} \ln y \right)^2 \, dy \)

(c) \( \int_0^\pi \left[ \frac{\left( \frac{1}{2} \ln 2 \right)^2}{2} \left( \frac{1}{2} \ln 2 - \frac{1}{2} \ln y \right)^2 \right] \, dy \) (d) \( \int_0^{\ln 2} \frac{\pi}{2} \left( 2 - e^{2x} \right)^2 \, dx \)

(4) (a) \( \int_0^3 \pi \left[ (10 - x^2)^2 - (10 - 3x)^2 \right] \, dx \) (b) \( \int_0^\pi \left[ \sqrt{y + 1}^2 - \left( \frac{y}{3} + 1 \right)^2 \right] \, dy \)

【6.3】Volume by Shells

(1) \( \int_0^\ln 2 2 \pi y \left( \frac{1}{2} \ln y \right) \, dy \) (b) \( \int_0^{\ln 2} 2 \pi x \left( 2 - e^{2x} \right) \, dx \) (2) (a) \( \int_0^\pi 2 \pi \left( x + 5 \right) x^2 \, dx \)

(b) \( \int_0^\pi 2 \pi \left( 5 - x \right) x^2 \, dx \) (c) \( \int_0^\pi 2 \pi \left( y + 5 \right) \left( 2 - \sqrt{y} \right) \, dy \) (d) \( \int_0^\pi 2 \pi \left( 5 - y \right) \left( 2 - \sqrt{y} \right) \, dy \)

(3) (a) \( \int_0^\pi 2 \pi \left( 10 - y \right) \left( \sqrt{y} - \frac{4}{3} \right) \, dy \) (b) \( \int_0^\pi 2 \pi \left( x + 1 \right) \left( 3x - x^3 \right) \, dx \)

【8.1】Arc Length

(1) The Mean Value Theorem (2) \( f \left( x \right) = -2x^2 + 7 \) and \( f \left( x \right) = 2x^2 + 3 \) (3) \( \frac{4}{3} \) (4) \( \frac{55}{3} \)

【8.2】Surface Area

(1) \( 4 \pi r^2 \) square units (2) \( \pi \left( e + 1 \right) \) (3) \( \frac{21}{2} \pi \) (4) \( 16 \pi \) (5) \( \frac{10}{3} \pi \)