1. Use analytical methods to evaluate each limit. If a limit does not exist, explain why. Show all work for full credit.

   a) \( \lim_{x \to 0} \frac{3}{x+1} - \frac{3}{x} \)

   b) \( \lim_{x \to \infty} \frac{5 + x - 4x^3}{2x^2 + 5x + e} \)

   c) \( \lim_{x \to 1} \frac{\ln(5x^3)}{2x^2 + 5} \)

   d) \( \lim_{x \to \infty} \left[ x \ln \left( 1 + \frac{3}{x} \right) \right] \)

   e) \( \lim_{x \to 0} \frac{1 - e^{2x}}{\ln(x + 1)} \)

2. Find the derivative of each function.

   a) \( g(x) = \frac{x^2}{\pi + 2} \)

   b) \( f(x) = \sin \left( \frac{1}{e^x} \right) \)

   c) \( y = e^{\ln 4} \)

   d) \( h(t) = \frac{5t^4}{\csc(t + 1)} \)

   e) \( y = \sec(e^{x^2}) \)

   f) \( h(x) = \tan \left( \sin(e^{3x}) \right) \)

   g) \( y = x^{\cos(\ln x)} \)

3. Evaluate the following integrals. Clearly show all work, including any variable substitutions.

   a) \( \int \left( \frac{3}{\sqrt{x}} + \frac{2}{x^2} - \csc^2 x - 7 \right) \, dx \)

   b) \( \int \frac{x}{\sqrt{4 + 3x^2}} \, dx \)

   c) \( \int \sin^5 x \cos x \, dx \)

   d) \( \int \frac{(\ln x)^k}{x} \, dx \) (k a constant)

4. Find the equation of the line tangent to the graph at the indicated value of \( x \).

   \( f(x) = \tan(2x) + 1 \quad x = \frac{\pi}{8} \)

5. Using the limit definition of the derivative, find the derivative of \( f(x) = 2x^2 - 9 \).

6. A water tank has the shape of an inverted right circular cone, with a base radius of 5 feet and a height of 20 feet. A valve at the bottom of the tank is opened and water flows out at a constant rate of \( 2 \text{ ft}^3/\text{min} \). At what rate is water level falling at the instant the water in the tank is 8 feet deep? Your solution should contain a well-labeled picture with defined variables, an equation that describes the situation, and all calculus and algebra needed to determine the answer. Your final answer should be stated as a complete sentence with proper units.

   Note: The volume of a right circular cone is \( V = \frac{\pi}{3} r^2 h \).

7. Because of its larger mass, acceleration due to gravity on the planet Jupiter is approximately \( 26 \text{ m/sec}^2 \). (take downward to be the negative direction in your work below)

   a) A stone dropped from a height of 500 meters. Using calculus techniques, determine how long it will remain aloft. Round your final answer to the nearest hundredth of a second.

   b) What is the speed of the stone (nearest hundredth) upon impact with the ground?