Do You Remember Your First Year of Calculus?

How long has it been since you’ve done a bunch of calculus problems from a first or second semester of calculus? You might feel a little rusty. MATH 2060 expects you to shake the rust off very quickly. To be ready for the course, please look at the Course Learning Objectives or Outcomes for MATH 1060 and MATH 1080 posted on their respective web sites. Once you have refreshed your memory, you should work through the following problems.

1. Find the limit, if it exists. If the limit does not exist, explain why it does not.

   (a) \( \lim_{x \to 4} \frac{x - 4}{x^2 - 3x - 4} \)  
   (b) \( \lim_{x \to 8^-} \frac{|x - 8|}{x - 8} \)  
   (c) \( \lim_{x \to 1} \frac{1 - \sqrt{x}}{x - 1} \)

   (d) \( \lim_{x \to 10^-} \ln(100 - x^2) \)  
   (e) \( \lim_{x \to \pi/2^+} \tan x \)  
   (f) \( \lim_{x \to -\infty} \frac{x^2 - 9}{2x - 6} \)

   (g) \( \lim_{x \to \infty} \arctan(x^3 - x) \)  
   (h) \( \lim_{x \to \infty} \frac{\cos^2 x}{x^2} \)

2. If \( 2x - 1 \leq f(x) \leq x^2 \) for \( 0 < x < 3 \), find \( \lim_{x \to 1} f(x) \).

3. Let \( f(x) = \begin{cases} \sqrt{-x} & \text{if } x < 0 \\ 3 - x & \text{if } 0 \leq x < 3 \\ (x - 3)^2 & \text{if } x > 3 \end{cases} \). Where is \( f \) discontinuous?

4. Find a number \( \delta \) such that \( |(7x - 27) - 8| < 0.5 \) whenever \( 0 < |x - 5| < \delta \).

5. Find a function \( f \) and a number \( a \) such that \( \lim_{h \to 0} \frac{(2 + h)^6 - 64}{h} = f'(a) \).

6. Compute \( f'(x) \).

   (a) \( f(x) = (x + 1)^5(x + 2)^4 \)  
   (b) \( f(x) = \left(x + \frac{1}{x^4}\right)^{\sqrt{2}} \)  
   (c) \( f(x) = \sqrt{x} \tan(2x) \)

   (d) \( f(x) = \frac{x}{\sqrt{x^2 - 2x}} \)  
   (e) \( f(x) = \frac{1}{\cos(x + \cos x)} \)  
   (f) \( f(x) = \ln(\csc^2 x) \)

   (g) \( f(x) = e^{\cos x} + \cos(e^x) \)  
   (h) \( f(x) = 3^{-x^2} \)  
   (i) \( f(x) = \sin^{-1}(e^{2x}) \)

   (j) \( f(x) = \int_0^{x^2} \frac{2}{\sqrt{4 + t^2}} \, dt \)

7. The volume of a cube is increasing at a rate of 10 cm\(^3\)/min. How fast is the surface area increasing when the length of an edge is 20 cm?

8. Find the absolute maximum and minimum values of \( f(x) = 10 + 27x - x^3 \) on the interval \([0, 4]\).

9. For the function \( f(x) = \frac{x^2 - 1}{x^3} \), find (a) the domain; (b) all critical numbers; (c) all local maximum and minimum values; (d) all intervals of increase and decrease; (e) all intervals of concavity; (f) all inflection points; (g) all vertical and horizontal asymptotes.
10. Find the dimensions of the rectangle of largest area that has its base on the x-axis and its other two vertices above the x-axis and lying on the parabola \( y = 8 - x^2 \).

11. Find \( f(x) \) if \( f''(x) = x^4 - 4x^2 + 3x - 2 \), \( f(0) = 0 \), and \( f'(1) = 1 \).

12. Evaluate the integral, if possible.
   
   (a) \( \int_{0}^{1} (1 - x)^9 \, dx \)  
   (b) \( \int \frac{\cos(1/x)}{x^2} \, dx \)  
   (c) \( \int_{0}^{2\pi} \left| \sin x \right| \, dx \)
   
   (d) \( \int x^4 \ln x \, dx \)  
   (e) \( \int \frac{\sin^3 x}{\cos x} \, dx \)  
   (f) \( \int xe^{7x} \, dx \)
   
   (g) \( \int x \tan^{-1}(x^2) \, dx \)  
   (h) \( \int \frac{x}{x + 1} \, dx \)  
   (i) \( \int_{0}^{2} \frac{1}{x - 1} \, dx \)

13. Find the average value of \( f(x) = x^2\sqrt{1 + x^3} \) on the interval \([0, 2]\).

14. Find the area of the region bounded by \( x - 2y + 7 = 0 \) and \( y^2 - 6y - x = 0 \).

15. Find the length of the curve \( y = \frac{x^3}{6} + \frac{1}{2x} \), where \( 1 \leq x \leq 2 \).

16. Find the volume of the solid obtained by rotating about the y-axis the region under the curve \( y = \frac{1}{1 + x^4} \) from \( x = 0 \) to \( x = 1 \).

17. Eliminate the parameter to find a Cartesian equation that describes the curve given by the parametric equations
   \[ x = 1 + e^{2t}, \quad y = e^t \]

18. Sketch the polar curve \( r = 3 - \sin \theta \).

19. Find a polar curve represented by the Cartesian equation \( x^2 + y^2 = 5 \).

20. Find the points of intersection of the curves \( r = 2 \) and \( r = 4 \cos \theta \).

21. Find the area that lies inside both \( r = \sin \theta \) and \( r = \cos \theta \).

22. Identify and sketch the conic section with equation \( 25x^2 - 4y^2 + 50x - 16y = 91 \). Be sure to label any vertices or asymptotes.