You are not permitted to use a calculator on any portion of this test. You are not allowed to use any textbook, notes, cell phone, laptop, PDA, or any technology on either portion of this test. All devices must be turned off while you are in the testing room.
During this test, any communication with any person (other than the instructor or his designated proctor) in any form, including written, signed, verbal, or digital, is understood to be a violation of academic integrity.
No part of this test may be removed from the testing room.

Read each question very carefully. In order to receive full credit for the free response portion of the test, you must:
1. Show legible and logical (relevant) justification which supports your final answer.
2. Use complete and correct mathematical notation.
3. Include proper units, if necessary.
4. Give exact numerical values whenever possible.

You have 90 minutes to complete the entire test.

On my honor, I have neither given nor received inappropriate or unauthorized information at any time before or during this test.

Student’s Signature: ________________________________________________

Do not write below this line.

<table>
<thead>
<tr>
<th>Free Response Problem</th>
<th>Possible Points</th>
<th>Points Earned</th>
<th>Free Response Problem</th>
<th>Possible Points</th>
<th>Points Earned</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td></td>
<td>5a</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td></td>
<td>5b</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>3ab</td>
<td>5</td>
<td></td>
<td>6</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>3cd</td>
<td>3</td>
<td></td>
<td>7</td>
<td>1</td>
<td></td>
</tr>
<tr>
<td>3ef</td>
<td>4</td>
<td></td>
<td>Free Response</td>
<td>49</td>
<td></td>
</tr>
<tr>
<td>3g</td>
<td>2</td>
<td></td>
<td>Multiple Choice</td>
<td>51</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td></td>
<td>Test Total</td>
<td>100</td>
<td></td>
</tr>
</tbody>
</table>
Multiple Choice. There are 17 multiple choice questions. Each question is worth 3 points and has one correct answer. The multiple choice problems will count 51% of the total grade. Use a number 2 pencil and bubble in the letter of your response on the scantron sheet for problems 1 – 17. For your own record, also circle your choice on your test since the scantron will not be returned to you. Only the responses recorded on your scantron sheet will be graded. You are NOT permitted to use a calculator on any portion of this test.

#1. Solve $5y^2xz + 2yx^2 = 2yx - 4xyz - y^2xz$ for $z$.

- a) $\frac{2}{4+y}$
- b) $\frac{1-2x}{2y+2x}$
- c) $\frac{-2x}{5y}$
- d) $\frac{1-x}{3y+2}$

#2. Determine $\lim_{x \to 1} \frac{x^2+3x-4}{x^2-1}$.

- a) does not exist
- b) $\frac{0}{0}$
- c) $\infty$
- d) $\frac{5}{2}$

#3. The solutions to $(a + b)x^2 - kx - y^2 = 0$ are:

- a) $x = \frac{k \pm \sqrt{k^2+4ay^2+4by^2}}{2a+2b}$
- b) $x = \frac{-k \pm \sqrt{-k^2-4ay^2+4by^2}}{2a}$
- c) $x = \frac{k \pm \sqrt{k^2+4ay^2+4by^2}}{2a}$
- d) $x = \frac{-k \pm \sqrt{-k^2-4ay^2+4by^2}}{2a+2b}$

#4. A continuous function $f(x)$ satisfies $f(-11) = 1$ and $f(-8) = -2$. What conclusion (if any) does the Intermediate Value Theorem allow us to draw?

- a) $f(x) = 0$ for some $x$ in the interval $(-2, 1)$
- b) We cannot draw a conclusion based on this information.
- c) $f(x) = 0$ for some $x$ in the interval $(-11, -8)$
- d) $f(x)$ does not cross the $x$-axis anywhere on the interval $(-11, 1)$. 
#5. Determine \( \lim_{h \to 0} \frac{2}{\sqrt{3h+4}+2} \).

- a) \( \infty \)
- b) 1
- c) \( \frac{1}{2} \)
- d) does not exist

#6. Determine all solutions to \( \frac{1}{2} |x + 3| < 7 \)

- a) no solution
- b) \( (-\infty, -17) \cup (11, \infty) \)
- c) \( (-17, 11) \)
- d) all real numbers

#7. Simplify completely: \( \sqrt{x^4 - 9x^2} \)

- a) \( x \cdot \sqrt{x^2 - 9} \)
- b) \( |x| \cdot |x - 3| \)
- c) \( x(x - 3) \)
- d) \( |x| \cdot \sqrt{x^2 - 9} \)

#8. Use factoring by grouping to find all solutions, both real and complex, to \( x^3 - x^2 + 4x = 4 \)

- a) \( x = 0, x = 1, x = 4 \)
- b) \( x = 1, x = \pm 2i \)
- c) \( x = 1, x = \frac{1}{2} \pm \frac{\sqrt{15}}{2} i \)
- d) \( x = 1 \)

#9. Rationalize the denominator: \( \frac{\sqrt{x}}{\sqrt{x} + \sqrt{y}} \)

- a) \( \frac{x - \sqrt{xy}}{x - y} \)
- b) \( \frac{\sqrt{xy}}{y} \)
- c) \( \frac{x}{x + \sqrt{xy}} \)
- d) \( \frac{x + \sqrt{xy}}{x + y} \)

#10. Evaluate \( \lim_{x \to 4^+} \frac{x - 4}{|4 - x|} \)

- a) \(-1\)
- b) 1
- c) \( \infty \)
- d) does not exist
#11. Determine the quotient and remainder when $4x^3 + 4x - 6$ is divided by $2x - 1$.

- **a)** Quotient: $2x^2 - x$; Remainder: $5x - 6$
- **b)** Quotient: $5x - 6$; Remainder: $2x^2 - x$
- **c)** Quotient: $5x - 6$; Remainder: $2x^2 + x$
- **d)** Quotient: $2x^2 + x$; Remainder: $5x - 6$

#12. Determine all points at which the function graphed below is discontinuous.

![Graph](image)

- **a)** $x = -2$
- **b)** the function is continuous everywhere
- **c)** $x = -2, x = 1$
- **d)** $x = 1$

#13. Given that $-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$ for all values of $x > 0$, what conclusion (if any) does the Sandwich Theorem allow us to draw about $\lim_{x \to 2} \frac{\sin x}{x}$?

- **a)** $\lim_{x \to 2} \frac{\sin x}{x} = \frac{1}{2}$
- **b)** $\lim_{x \to 2} \frac{\sin x}{x}$ does not exist.
- **c)** $\lim_{x \to 2} \frac{\sin x}{x} = \frac{1}{2}$
- **d)** We can draw no conclusion.

#14. Determine all solutions to $\frac{3x}{x-3} - \frac{4}{x} = \frac{12}{x^2-3x}$.

- **a)** $0, \frac{4}{3}$
- **b)** $\frac{4}{3}$
- **c)** $\frac{3}{4}$
- **d)** $-\frac{3}{4}, \frac{3}{4}$
#15. Use the graph shown to find a $\delta > 0$ such that for all $x$, $0 < |x - x_0| < \delta \Rightarrow |f(x) - L| < \varepsilon$.

\[ f(x) = \sqrt{2x} \]
\[ x_0 = 5 \]
\[ L = \sqrt{10} \]
\[ \varepsilon = \frac{1}{4} \]

![Graph of $f(x) = \sqrt{2x}$](graph.png)

a) 2.91   b) 0.8213   c) 4.2412   d) 0.7588

#16. Determine $\lim_{x \to 6} \frac{x+6}{x-6}$.

a) $-1$   b) $-\infty$   c) does not exist   d) $\infty$

#17. Determine $\lim_{x \to -\infty} \frac{-12x^2+8x+9}{19+6x-2x^2}$.

a) $\frac{9}{19}$   b) $-\frac{12}{19}$   c) $\infty$   d) 6
Free Response. The Free Response questions will count 49% of the total grade. Read each question carefully. In order to receive full credit you must show legible and logical (relevant) justification which supports your final answer. Give answers as exact answers. You are NOT permitted to use a calculator on any portion of this test.

1. (10 points) Use the graph of \( f(x) \) below to find each of the following limits. Use correct notation in reporting your answers.

![Graph of f(x)](image)

a. \( \lim_{x \to -2^-} f(x) = 5 \)

b. \( \lim_{x \to -2^+} f(x) = -3 \)

c. \( \lim_{x \to -2} f(x) \text{ DNE} \)

d. \( \lim_{x \to 2^-} f(x) = -4 \)

e. \( \lim_{x \to 2^+} f(x) = -4 \)

f. \( \lim_{x \to 2} f(x) = -4 \)

g. \( \lim_{x \to 6^-} f(x) = 2 \)

h. \( \lim_{x \to 6^+} f(x) = 2 \)

i. \( \lim_{x \to 6} f(x) = 2 \)

j. \( \lim_{x \to -6^+} f(x) = -4 \)

Deduct 1 point for more than 3 missing or extra “=”

Deduct 1 point for more than 3 missing for extra “=”

2. (6 points) Given \( f(x) = \begin{cases} \frac{x^2}{x+k}, & \text{if } x \leq 4 \\ x^2, & \text{if } x > 4 \end{cases} \)

a. Find \( \lim_{x \to 4^-} f(x) \) and \( \lim_{x \to 4^+} f(x) \). (Show the appropriate function for each.)

\[
\lim_{x \to 4^-} f(x) = \lim_{x \to 4^-} \frac{x^2}{x+k} = 4^2 = 16
\]

\[
\lim_{x \to 4^+} f(x) = \lim_{x \to 4^+} (x+k) = 4 + k
\]
b. Evaluate \( f(4) \).

\[
f(4) = 4^2 = 16
\]

1 point A/N

---

b. (2 points) Using limits, show that \( f(x) \) has a removable discontinuity at \( x = -2 \).

(We can define \( f(-2) \) in a way that extends \( f(x) \) to be continuous at \( x = -2 \). Find this extended value for \( f(-2) \) using limits.)

\[
\lim_{x \to -2^-} f(x) = \lim_{x \to -2^-} \frac{x-2}{x-1} = \frac{-4}{-3} = \frac{4}{3}
\]

\[
\lim_{x \to -2^+} f(x) = \lim_{x \to -2^+} \frac{x-2}{x-1} = \frac{-4}{-3} = \frac{4}{3}
\]

---

### Work on problem

| Substitute correct piece of function in left-hand limit | 1 |
| Substitute correct piece of function in right-hand limit | 1 |
| Correct evaluation of left-hand limit | 1 |
| Correct evaluation of right-hand limit | 1 |

Notes: MUST show function used for each limit
Correct evaluation of wrong limit (e.g. using \( x+k \) for right hand limit but evaluating correctly) receives 1 out of 2 points
Work on problem | Points Awarded
---|---
Examining limit from the left as $x \to -2$ | $\frac{1}{2}$
Examining limit from the right as $x \to -2$ | $\frac{1}{2}$
correct left-hand limit evaluation using simplified form of $f(x)$ | $\frac{1}{2}$
correct right-hand limit evaluation using simplified form of $f(x)$ | $\frac{1}{2}$
Notes: $\frac{1}{2}$ point deduction for improper use of notation

c. (2 point) Find the $x$-intercept(s) for $f(x)$. Write the intercept(s) as ordered pair(s).

Zeroes of the numerator are $-2$ and $2$ but $f(-2)$ is undefined since $-2$ is not in the domain of $f(x)$. The only $x$-intercept is at $(2,0)$.

Work on problem | Points Awarded
---|---
Finding zeroes of the numerator | $\frac{1}{2}$
Writing intercept as ordered pair $(2, 0)$ | 1
Excluding the zero at $-2$ since it’s not in the domain | $\frac{1}{2}$
Notes:

d. (1 point) Find the $y$-intercept for $f(x)$. Write the intercept as an ordered pair.

$$f(0) = \frac{0^2 - 4}{0^2 + 0 - 2} = \frac{-4}{-2} = 2$$

The $y$-intercept is at $(0, 2)$.

Work on problem | Points Awarded
---|---
Finding f(0) | $\frac{1}{2}$
Writing intercept as ordered pair $(0, 2)$ | $\frac{1}{2}$
Notes:

e. (2 points) Does $f(x)$ have a horizontal asymptote? If so, find it and give the equation for the horizontal asymptote. Use a limit argument to support your answer. If not, state why not.

$$\lim_{x \to \infty} f(x) = \lim_{x \to \infty} \frac{x^2 - \frac{4}{x^2}}{x^2 + \frac{x}{x^2} - \frac{2}{x^2}} = \lim_{x \to \infty} \frac{1 - 4 \lim_{x \to \infty} \frac{1}{x^2}}{\lim_{x \to \infty} 1 + \lim_{x \to \infty} \frac{1}{x^2} - 2 \lim_{x \to \infty} \frac{1}{x^2}} = \frac{1 - 4 \cdot 0}{1 + 0 - 2 \cdot 0} = 1$$

OR

$$\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} \frac{x^2 - \frac{4}{x^2}}{x^2 + \frac{x}{x^2} - \frac{2}{x^2}} = \lim_{x \to -\infty} \frac{1 - 4 \lim_{x \to -\infty} \frac{1}{x^2}}{\lim_{x \to -\infty} 1 + \lim_{x \to -\infty} \frac{1}{x^2} - 2 \lim_{x \to -\infty} \frac{1}{x^2}} = \frac{1 - 4 \cdot 0}{1 + 0 - 2 \cdot 0} = 1$$

YES, there is a horizontal asymptote at $y = 1$.  

Page 8 of 12
Work on problem | Points Awarded
---|---
Investigating limit as $x \to \infty$ or as $x \to -\infty$ | $\frac{1}{2}$
Correct evaluation of limit | $\frac{1}{2}$
Correct conclusion | $\frac{1}{2}$
Equation of horizontal asymptote | $\frac{1}{2}$

Notes:
1) Okay to use simplified form
2) Do NOT have to investigate limit at $\infty$ AND at $-\infty$; one is sufficient
3) $\frac{1}{2}$ point deduction for improper use of notation

f. (2 points) Does $f(x)$ have a slant/oblique asymptote? If so, find it and give the equation for the oblique asymptote. You do not need to use a limit argument. If not, state why not.

No, there is no slant asymptote. The function has a horizontal asymptote and a rational function cannot have both a slant asymptote AND a horizontal asymptote.

OR

No, there is no slant asymptote. The degree of the numerator equals the degree of the denominator.

OR

No, there is no slant asymptote. Polynomial long division results in a constant quotient, not a linear quotient.

Work on problem | Points Awarded
---|---
Correct conclusion | 1
Valid reasoning | 1

Notes:

(2 points) Use the information you have gathered to graph $f(x)$. **Plot all asymptotes and any intercepts. Clearly show any discontinuities within the graph.** This sketch will be right or wrong. It will not be graded correct based on incorrect answers in parts (a) – (f).
4. (4 points) Factor completely: \( 27m^4 - m \)

\[
27m^4 - m = m(27m^3 - 1) = m(3m - 1)(9m^2 + 3m + 1)
\]

<table>
<thead>
<tr>
<th>Work on problem</th>
<th>Points Awarded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factoring out largest common factor of m</td>
<td>1</td>
</tr>
<tr>
<td>Recognizing difference of cubes</td>
<td>1</td>
</tr>
<tr>
<td>Correct linear factor in difference of cubes (2m-1)</td>
<td>1</td>
</tr>
<tr>
<td>Correct quadratic factor in difference of cubes (4m^2+2m+1)</td>
<td>1</td>
</tr>
</tbody>
</table>

Notes: ½ point deduction for sign or squaring error in quadratic factor.
5. (4 points each) Evaluate each of the following limits, showing all work and reasoning:

a. \( \lim_{x \to 0} \frac{\sin 5x}{2x} \)

\[
\lim_{x \to 0} \frac{\sin 5x}{2x} = \frac{1}{2} \lim_{x \to 0} \frac{\sin 5x}{x} = \frac{1}{2} \lim_{x \to 0} \frac{5 \sin 5x}{5x} = \frac{5}{2} \lim_{x \to 0} \frac{\sin 5x}{5x} = \frac{5}{2} \cdot 1 = \frac{5}{2}
\]

**Work on problem**

<table>
<thead>
<tr>
<th>Points Awarded</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pulling constant multiples of ( \frac{1}{2} ) and 5 (or of 5/2 in one step) out of limit</td>
</tr>
<tr>
<td>Multiplying by “1” in the form of 5/5 so that the denominator matches the argument of the sine function</td>
</tr>
<tr>
<td>Evaluating ( \lim_{x \to 0} \frac{\sin 5x}{5x} = 1 )</td>
</tr>
</tbody>
</table>

**Notes:**

1) \( \frac{1}{2} \) point deduction for improper use of notation
2) 1 point awarded for evaluating \( \lim_{x \to 0} \frac{\sin x}{x} = 1 \) after incorrectly stating \( \sin(5x) = 5 \sin x \)

b. \( \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \)

\[
\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} = \lim_{x \to 16} \frac{\sqrt{x} - 4}{(\sqrt{x} + 4)(\sqrt{x} - 4)} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} = \frac{1}{16 + 4} = \frac{1}{20} = \frac{1}{8}
\]

OR

\[
\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} = \lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x} + 4}{\sqrt{x} + 4} = \lim_{x \to 16} \frac{x - 16}{(\sqrt{x} + 4)(\sqrt{x} - 16)(\sqrt{x} + 4)} = \lim_{x \to 16} \frac{1}{\sqrt{x} + 4} = \frac{1}{8}
\]

**Work on problem**

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Multiplying by conjugate OR factoring denominator</td>
</tr>
<tr>
<td>Cancelling common factor</td>
</tr>
<tr>
<td>Correct evaluation of limit using direct substitution after cancelling common factor</td>
</tr>
</tbody>
</table>

**Notes:**

1) \( \frac{1}{2} \) point deduction for improper use of notation
2) 1 point awarded for setting up multiplication by conjugate correctly but either multiplying incorrectly or expanding denominator so that cancellation of common factors became impossible
6. (6 points) Consider the function $f(x) = \sqrt{x - 1}$. We know that $\lim_{x \to 5} f(x) = 2$. **Algebraically** find the greatest value for $\delta > 0$ such that for all $x$ satisfying $0 < |x - 5| < \delta$, the inequality $|f(x) - 2| < \frac{1}{2}$ holds.

$$|\sqrt{x - 1} - 2| < \frac{1}{2}$$

$$-\frac{1}{2} < \sqrt{x - 1} - 2 < \frac{1}{2}$$

$$\frac{3}{2} < \sqrt{x - 1} < \frac{5}{2}$$

$$\frac{9}{4} < x - 1 < \frac{25}{4}$$

$$\frac{13}{4} < x < \frac{29}{4}$$

$$5 - \delta_1 = \frac{13}{4} \quad \Rightarrow \quad \delta_1 = 5 - \frac{13}{4} = \frac{7}{4}$$

$$5 + \delta_2 = \frac{29}{4} \quad \Rightarrow \quad \delta_2 = \frac{29}{4} - 5 = \frac{9}{4}$$

Choose $\delta$ to be the smaller of $\delta_1, \delta_2$ so $\delta = \frac{7}{4}$.

<table>
<thead>
<tr>
<th>Work on problem</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Substituting $\sqrt{x - 1}$ for $f(x)$ in the absolute value inequality</td>
<td>1</td>
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<tr>
<td>Converting the absolute value inequality to two inequalities not involving</td>
<td>1</td>
</tr>
<tr>
<td>absolute values</td>
<td></td>
</tr>
<tr>
<td>Adding 2 to both sides of both inequalities</td>
<td>1</td>
</tr>
<tr>
<td>Squaring both sides of both inequalities</td>
<td>1</td>
</tr>
<tr>
<td>Adding one to both sides of both inequalities</td>
<td>1</td>
</tr>
<tr>
<td>Determining $\delta_1, \delta_2$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Choosing $\delta$ to be the smaller of $\delta_1, \delta_2$</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
<td>Notes:</td>
<td></td>
</tr>
</tbody>
</table>

7. (1 pt) Check to make sure your Scantron form … (deleted for space)